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Exponential Similarity Measures Under q-Rung Orthopair Spherical Fuzzy Environment Application to Multi- Criteria Decision- Making

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ABSTRACT

Q-rung orthopair spherical fuzzy sets (q-RO-SFSs) extend the expressive capability of fuzzy models by incorporating satisfaction, non-satisfaction, neutrality, and hesitancy degrees under a flexible q-rung structure. In this paper, we develop a family of exponential similarity measures for q-RO-SFSs, designed to capture complex uncertainty with improved sensitivity to all four components. Weighted, average, and generalized forms of the measures are proposed, and their mathematical validity is established. The applicability of the method is illustrated through multi-criteria decision-making problems, including credit evaluation and pattern recognition. Comparative analysis with existing similarity-based approaches shows that the proposed measures provide more stable and reliable rankings, demonstrating their effectiveness for practical decision-making in uncertain environments.

1. Introduction

Decision-making plays a vital role in identifying optimal solutions in complex real-life scenarios where multiple conflicting criteria are involved. In most situations, decision-makers face incomplete, vague, or imprecise information due to uncertainty in human judgments or limitations in available data. This makes it difficult to arrive at accurate or consistent conclusions. To address such uncertainty, fuzzy set theory (FST), introduced by Zadeh [1], provides a mathematical foundation for modeling vagueness by assigning each element a membership degree (MD) between 0 and 1. However, classical fuzzy sets do not explicitly represent hesitancy in assigning membership values.

To overcome this limitation, Atanassov [2] proposed intuitionistic fuzzy sets (IFSs), which include both membership and non-membership degrees, with hesitancy implicitly derived. IFSs provide a more complete framework and have been widely applied in decision-making. However, in

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certain contexts such as human voting, machine vision, and medical diagnosis, FSs and IFSs may be insufficient. To handle these cases, Cuong and Kreinovich[3] proposed picture fuzzy sets (PFSs), characterized by satisfaction, non-satisfaction, and neutrality degrees, offering closer alignment with human decision behavior.

To further enhance flexibility, Ashraf et al. [4] introduced spherical fuzzy sets (SFSs), which allow three independent degrees satisfaction, non-satisfaction, and neutrality subject to the condition $0 \leq \ {a_{x_1}}^2(b_t) + {b_{x_1}}^2(b_t) + {c_{x_1}}^2(b_t) \leq 1$. SFSs generalize several fuzzy set models and have been shown to produce more precise decision outcomes. Mahmood et al. [5] contributed foundational developments in SFS theory, and numerous similarity measures (SMs) have been developed for IFSs, PFSs, and SFSs [6–19]. These include cosine measures [6, 26], set pair analysis approaches [7], Jaccard index-based clustering [8], exponential distance measures [9], aggregation operators [23], correlation coefficients [24], and SM-based algorithms [25]. Variants such as interval-valued PFSs [27], hesitant PFSs [28, 29], and linguistic PFSs [30] have been successfully applied to domains such as healthcare [31] and site selection [32]. Additional contributions include accuracy functions [33, 34], operations [35], and improved score functions [36,37].

Recently, the q-rung orthopair fuzzy set (q-ROFS), introduced by Yager [43] and extended by Liu and Wang [44], has emerged as a powerful generalization of IFSs and PFSs. In q-ROFSs, the truth membership degree a_{x_1} and falsity non-membership degree b_{x_1}satisfy:

$$a_{x_1}^q + b_{x_1}^q \leq 1$$

where $q \ \geq \ 1$is an adjustable parameter, for $q \ = \ 1$, the model reduces to an IFS; for $q \ = \ 2$, it becomes a PFS. Larger values of q allow greater flexibility, permitting higher simultaneous values of $a_{x_1}(b_t)$and$b_{x_1}(b_t)$, and enabling richer representation of uncertainty, neutrality, and hesitancy in expert judgments. This makes q-ROFSs particularly suitable for complex multi-criteria decision-making (MCDM) problems [43,44].

Despite the advantages of q-ROFSs and SFSs individually, their integration has received limited attention. Most existing SFS similarity measures are based on Hamming or cosine distances [40–42], which may not fully capture the independent effects of membership and non-membership degrees or adequately incorporate hesitancy. Furthermore, studies have shown that exponential similarity measures can enhance sensitivity and discrimination ability in fuzzy decision-making by amplifying deviations between compared elements [9,48]. Such measures can offer more nuanced comparisons, especially in environments involving multiple uncertainty components. Motivated by these observations, this paper proposes a novel family of exponential similarity measures for q-rung orthopair spherical fuzzy sets (q-RO-SFSs). The proposed measures incorporate satisfaction, non-satisfaction, neutrality, and hesitancy in weighted, average, and generalized forms to handle varying importance of criteria. The mathematical properties of the measures are formally proven. Their applicability is demonstrated through MCDM problems such as food waste treatment technology selection, credit evaluation, and pattern recognition. Comparative analysis with existing approaches confirms that the proposed methods provide more stable and reliable rankings in uncertain environments.

This paper focuses on developing exponential similarity measures for q-rung orthopair spherical fuzzy sets. The work covers the mathematical formation of the proposed measures, including weighted, average, and generalized versions. Their basic properties are verified to ensure consistency within the spherical fuzzy environment. The study also applies these measures to multi-criteria decision-making problems, with a detailed demonstration using food waste treatment technology selection. The results are compared with existing similarity-based methods to show how the proposed approach performs in practical situations involving uncertainty.

The main contributions of this work are summarized as follows:

(i) A novel family of exponential similarity measures is developed for q-rung orthopair spherical fuzzy sets;

(ii) Weighted, average, and generalized similarity models are formulated to handle multi-criteria decision-making problems with varying importance of criteria;

(iii) The proposed similarity measures are mathematically validated by verifying essential properties such as boundedness, symmetry, identity, and monotonicity;

(iv) The applicability of the proposed approach is demonstrated through real-world decision-making problems, including food waste treatment technology and medical waste management;

(v) The effectiveness and robustness of the proposed method are established through comparative analysis and sensitivity analysis.

2. Preliminaries

In this section, we briefly review the basic concepts related to SFS and SM over the set B. Suppose, a set of collection of all fuzzy sets in the universe is $B = \{b_1, b_2, b_{3,} \dots, b_p\}$.

Definition 1: [1] A fuzzy set X_1 in B is givenby

$$X_1 = \{(b_t, a_{x_1}(b_t)) \mid b_t \in B\}$$

where$a_{x_1}(b_t) \in [0,1]$ signify the membership degree for any b_t to the setX_1, $t = 1,2, \dots, p$.

Definition 2: [2] A set X_1 in B defined as

$$X_1 = \{(b_t, a_{x_1}(b_t), b_{x_1}(b_t)) \mid x \in B\}$$

is said to be intuitionistic fuzzy set, where $a_{x_1}(b_t)$, $b_{x_1}(b_t) \in [0,1]$, and satisfy the valuable condition

$0 \leq a_{x_1}(b_t) + b_{x_1}(b_t) \leq 1 \,\forall\, b_{x_1} \in B$.

Here $a_{x_1}(b_t)$, $b_{x_1}(b_t) \in [0, 1]$ are called degree of membership function and non-membership function of B toX_1, respectively. So, set X_1 can be rewritten as

$$X_1 = \{(b_t, a_{x_1}(b_t), 1 - a_{x_1}(b_t)) \mid a_{x_1} \in B\}.$$

Also

$d_{x_1}(b_t) = 1 - a_{x_1}(b_t) - b_{x_1}(b_t)$.

Obviously, $0 \leq d_{x_1}(b_t) \leq 1$ for each $b_t \in X_1$ and $d_{x_1}(b_t)$ is the hesitancy degree of B to X_1.

Definition 3: [6] A spherical fuzzy set X_1 in B is given by

$$X_1 = \left\{\left(b_t, a_{x_1}(b_t), b_{x_1}(b_t), c_{x_1}(b_t)\right), b_t \in B\right\},$$

where $0 \leq a_{x_1}(b_t) \leq 1$, $0 \leq b_{x_1}(b_t) \leq 1$ and$0 \leq c_{x_1}(b_t) \leq 1$ are assessment level of satisfaction, level of non- satisfaction and neutral level respectively for any $b_t \in B$ inX_1 such that $0 \leq a_{x_1}{}^2(b_t) + b_{x_1}{}^2(b_t) + c_{x_1}{}^2(b_t) \leq 1$. Also$d_{c_1} = \sqrt{1 - a_{x_1}{}^2(b_t) - b_{x_1}{}^2(b_t) - c_{x_1}{}^2(b_t)}$ represent level of refusal degree for$b_t \in B$ in X_1.

Definition 4: [34-35] In q-rung orthopair fuzzy set (q-ROFS) an object X_1 in the universe set B have the representation

$$X_1 = \left\{\left(a_{x_1}(b_t), b_{x_1}(b_t)\right) | b_t \in B\right\}.$$

Here the function $a_{x_1}: X_1 \to [0,1]$ defines the truth membership function and $b_{x_1}: X_1 \to [0,1]$ defines the falsity membership functionof the element$b_t \in B$ for every $b_t \in B$, where

$$0 \leq a_{x_1}^q + b_{x_1}^q \leq 1, q \geq 1.$$

Furthermore, $d_{x_1}(b_t) = \sqrt[q]{1 - a_{x_1}^q(b_t) - b_{x_1}^q(b_t)}$ is called the q- ROF index of b_t to setX_1.

Definition 5: [45] Let $S = (a_x, b_x, c_x, d_x)$, $S_1 = \left(a_{x_1}, b_{x_1}, c_{x_1}, d_{x_1}\right)$and $S_2 = \left(a_{x_2}, b_{x_2}, c_{x_2}, d_{x_2}\right)$ be three SFNs, then we have

(i) $S^c = (a_x, b_x, c_x, d_x)$

(ii) S1⊆S2 if and only if,$a_{x_1} \geq a_{x_2}$, $b_{x_1} \geq b_{x_2}$,$c_{x_1} \geq c_{x_2}$,$d_{x_1} \geq d_{x_2}$

(iii) S1 = S2 if all four components are equal

(iv)S1 ∩ S2 = $(min\ (a_{x_1}, a_{x_2}), max\ (b_{x_1}, b_{x_2}), max\ (c_{x_1}, c_{x_2}), max\ (d_{x_1}, d_{x_2}))$

(v) S1∪S2 = $(max\ (a_{x_1}, a_{x_2}), min\ (b_{x_1}, b_{x_2}), min\ (c_{x_1}, c_{x_2}), min\ (d_{x_1}, d_{x_2}))$.

Definition 6: [46]Let Φ_s(X) be the collection of all Spherical Fuzzy Sets (SFSs) over a finite universe B. A function$S: \Phi_s(\mathrm{X}) \times \Phi_s(\mathrm{X}) \to [0, 1]$is called a similarity measure between two SFSs {$P = \left\{\langle b_t, a_{x_1}(b_t), a_{x_1}(b_t), c_{x_1}(b_t), d_{x_1}(b_t)\rangle | b_t \in B\right\}$and

$Q = \left\{\langle b_t, a_{x_2}(b_t), a_{x_2}(b_t), c_{x_2}(b_t), d_{x_2}(b_t)\rangle | b_t \in B\right\}$,if following properties hold:

(P1) $0 \leq S(P, Q) \leq 1$

(P2) $S\ (P,Q)\ =\ 1\ \Leftrightarrow\ P\ =\ Q$

(P3) $S\ (P,Q)\ =\ S\ (Q,P)$

(P4) If $P\ \subseteq\ Q\ \subseteq\ R$, then $S(P,R)\ \leq\ min\{S\ (P,Q), \mathrm{S\ (Q,R)}\}$ where $P,Q,R\ \in \Phi_S(X)$.

3. New q- rung based similarity measures on Spherical fuzzy sets

In this section we present q- rung SM-based on exponential functions for MDs and NMDs under SFS environment over the finite set B.

Definition 7: Consider two SFSs $P = \{\langle b_t, a_{x_2}(b_t), a_{x_2}(b_t), c_{x_2}(b_t), d_{x_2}(b_t)\rangle | b_t \in B\}$and

$Q = \{\langle b_t, a_{x_2}(b_t), a_{x_2}(b_t), c_{x_2}(b_t), d_{x_2}(b_t)\rangle | b_t \in B\}$.The four exponential functions are defined as

$$S_t^a(P,Q) = e^{-\left|a_P^q(b_t)-a_Q^q(b_t)\right|} \tag{1}$$

$$S_t^b(P,Q) = e^{-\left|b_P^q(b_t)-b_Q^q(b_t)\right|} \tag{2}$$

$$S_t^c(P,Q) = e^{-\left|c_P^q(b_t)-c_Q^q(b_t)\right|} \tag{3}$$

$$S_t^d(P,Q) = e^{-\left|d_P^q(b_t)-d_Q^q(b_t)\right|} \tag{4}$$

Theorem 1: For two spherical fuzzy set,$P = \{\langle b_t, a_{x_2}(b_t), a_{x_2}(b_t), c_{x_2}(b_t), d_{x_2}(b_t)\rangle | b_t \in B\}$and $Q = \{\langle b_t, a_{x_2}(b_t), a_{x_2}(b_t), c_{x_2}(b_t), d_{x_2}(b_t)\rangle | b_t \in B\}$

We have

$(P1) 0 \leq S_t^a(P,Q), S_t^b(P,Q), S_t^c(P,Q), S_t^d(P,Q) \leq 1;$

$(P2)\ S_t^a\ (P,Q) = S_t^a(Q,P), S_t^b(P,Q) = S_t^b(Q,P), S_t^c(P,Q) =\ S_t^c(Q,P), S_t^d(P,Q) =\ S_t^d(Q,P);$

$(P3)\ S_t^a\ (P,Q)\ =\ S_t^b\ (P,Q)\ =\ S_t^c\ (P,Q)\ =\ S_t^d\ (P,Q)\ =\ 1\ if\ and\ only\ if\ P\ =\ Q;$

$$(P4)\ if\ P\ \subseteq\ Q\ \subseteq\ R, then\ S_t^a(P,R)\ \leq\ min\{S_t^a\ (P,Q)\ , S_t^a(Q,R)\}, S_t^b(P,R) \leq\ min\{S_t^b(P,Q)\ , S_t^b(Q,R), S_t^c(P,R)\ \leq\ min\{S_t^c(P,Q)\ \ , S_t^c(P,R)\}\ and\ \ S_t^d(P,R) \leq\ min\{S_t^d\ (P,Q)\ ,\ S_t^d\ (Q,R)\}.$$

Proof: Let $P = a_{x_1}(b_t), b_{x_1}(b_t), c_{x_1}(b_t), d_{x_1}(b_t)$ and $Q = a_{x_2}(b_t), b_{x_2}(b_t), c_{x_2}(b_t), d_{x_2}(b_t)$be two SFSs over B.

(P1) By definition of SFSs, we have $a_P^{\mathrm{q}}(b_i) + b_Q^{\mathrm{q}}(b_i) + c_P^{\mathrm{q}}(b_i) + d_Q^{\mathrm{q}}(b_i) \leq 1$(q=2) for all bi∈ B. and similarly for Q

Thus, we have

$$-1 \leq -\left|a_P^q(b_t) - a_Q^q(b_t)\right| \leq 0$$

$$-1 \leq -\left|b_P^q(b_t) - b_Q^q(b_t)\right| \leq 0$$

$$-1 \leq -\left|c_P^q(b_t) - c_Q^q(b_t)\right| \leq 0$$

And

$$-1 \leq -\left|d_P^q(b_t) - d_Q^q(b_t)\right| \leq 0$$

Hence

$$0 \leq e^{-\left|a_P^q(b_t) - a_Q^q(b_t)\right|} \leq 1,$$

$$0 \leq e^{-\left|b_P^q(b_t) - b_Q^q(b_t)\right|} \leq 1,$$

$$0 \leq e^{-\left|c_P^q(b_t) - c_Q^q(b_t)\right|} \leq 1$$

And

$$0 \leq e^{-\left|d_P^q(b_t) - d_Q^q(b_t)\right|} \leq 1$$

Thus,

(P1) holds.

(P2) It is obtained from the definition.

(P3) if$S_t^a(P,Q) = S_t^b(P,Q) = S_t^c(P,Q) = S_t^d(P,Q) = 1$,

Then

$a_P^{\mathrm{q}}(b_t) = a_Q^{\mathrm{q}}(b_t)$,$b_P^{\mathrm{q}}(b_t) = b_Q^{\mathrm{q}}(b_t)$, $c_P^{\mathrm{q}}(b_t) = c_Q^{\mathrm{q}}(b_t)andd_P^{\mathrm{q}}(b_t) = d_Q^{\mathrm{q}}(b_t)$

for all b_t∈ B. It means that P = Q. On the other hand, if P = Q, then it is clearly gives

$S_t^a(\mathrm{P},\mathrm{Q})$ = $S_t^b(\mathrm{P},\mathrm{Q})$ = $S_t^c(\mathrm{P},\mathrm{Q})$ =$S_t^d(\mathrm{P},\mathrm{Q}) = 1$.

(P4) If P ⊆ Q ⊆ R, then for b_t∈ B, we have

$$0 \leq a_P(b_t) \leq a_Q(b_t) \leq a_R(b_t) \leq 1$$

$$1 \geq b_P(b_t) \geq b_Q(b_t) \geq b_R(b_t) \geq 0$$

$$1 \geq c_P(b_t) \geq c_Q(b_t) \geq c_R(b_t) \geq 0$$

And

$$1 \geq \ d_P(b_t) \geq d(b_t) \geq \ d_R(b_t) \geq \ 0.$$

This implies that (q=2)

$$0 \ \leq a^q{}_P(b_t) \ \leq a^q{}_Q(b_t) \ \leq \ a^q{}_R(b_t) \ \leq \ 1$$

$$1 \ \geq \ b^q{}_P(b_t) \ \leq b^q{}_Q(b_t) \ \leq \ b^q{}_R(b_t) \ \geq \ 0$$

$$1 \ \geq \ c^q{}_P(b_t) \ \leq c^q{}_Q(b_t) \ \leq \ c^q{}_R(b_t) \ \geq \ 0$$

And

$$1 \ \geq \ d^q{}_P(b_t) \ \leq d^q{}_Q(b_t) \ \leq \ d^q{}_R(b_t) \ \geq \ 0$$

Hence

$$-|a^q{}_P(b_t) - a^q{}_R(b_t)| \ \leq \ min\,\{-|a^q{}_P(b_t) - a^q{}_Q(b_t)|, -|a^q{}_Q(b_t) - a^q{}_R(b_t)|\}$$

$$-|b^q{}_P(b_t) - b^q{}_R(b_t)| \ \leq \ min\,\{-|b^q{}_P(b_t) - b^q{}_Q(b_t)|, -|b^q{}_Q(b_t) - b^q{}_R(b_t)|\}$$

$$-|c^q{}_P(b_t) - c^q{}_R(b_t)| \ \leq \ min\,\{-|c^q{}_P(b_t) - c^q{}_Q(b_t)|, -|c^q{}_Q(b_t) - c^q{}_R(b_t)|\}$$

And

$$-|d^q{}_P(b_t) - d^q{}_R(b_t)| \ \leq \ min\,\{-|d^q{}_P(b_t) - d^q{}_Q(b_t)|, -|d^q{}_Q(b_t) - d^q{}_R(b_t)|\}$$

It means that

$$S_t^a(P,R) \ \leq \ min\,\{S_t^a(P,Q), S_t^a(Q,R)\}$$

$$S_t^b(P,R) \ \leq \ min\,\{S_t^b\ (P,Q), S_t^b\ (Q,R)\}$$

$$S_t^c(P,R) \ \leq \ min\,\{S_t^c\ (P,Q), S_t^c\ (Q,R)\}$$

And

$$S_t^d(P,R) \ \leq \ min\,\{S_t^d\ (P,Q), S_t^d\ (Q,R)\}.$$

Next, Based on the two functions defined in Eqs. (1), (2), (3) and (4), we define the weighted SMs for SFSs as below.

Definition 8: Let P, Q be two SFSs defined over B and ω_i>0 is the weight of the element of B which satisfy$\sum_{i=1}^{n} \omega_i = 1$. Then, weighted SMs between them are defined as $S_x(P,Q) \ = \sum_{i=1}^{n} \omega_i \times \ S_t^a(P,Q) \ \times \ S_t^b(P,Q) \ \times \ S_t^c(P,Q) \ \times \ S_t^d(P,Q)$ .... (5)

Theorem 2: The measure defined in Definition (8) is a valid SMfor SFSs.

Proof: For two SFSs P and Q and from the Theorem 1, we have

(P1) Since $0 \ \leq \ S_t^a\ (P,Q), S_t^b\ (P,Q), S_t^c(P,Q), S_t^d(P,Q) \ \leq \ 1$ which implies that $0 \ \leq \ S_x(P,Q) \ \leq \sum_{i=1}^{n} \omega_i \ = \ 1$.

(P2) AsS_t^a, S_t^b, S_t^c and S_t^d are symmetrical for SFSs, so S_x also have this property.

(P3) As$S_t^a(P,Q) = S_t^b(P,Q), = (P,Q), = S_t^d(P,Q) = 1$ if and only if P = Q, so we get$S_x(P,Q) = 1$ if only if P = Q, because$\sum_{i=1}^{n} \omega_i = 1$.

(P4) For three PFSs P, Q and R satisfying P ⊆ Q ⊆R, we observed from Theorem 1 that $S_t^a(P,R) \leq min\{S_t^a(P,Q), S_t^a(Q,R)\}, S_t^b(P,R) \leq min\{S_t^b(P,Q), S_t^b(Q,R)\}$, $S_t^c(P,R) \leq min\{S_t^c(P,Q), S_t^c(Q,R)\}, S_t^d(P,R) \leq min\{S_t^d(P,Q), S_t^d(Q,R)\}$. Thus, based on it, Eq. (5) become

$$S_x(P,R) \leq S_x(P,Q) \; and \; S_x(P,R) \leq S_x(Q,R).$$

Besides this, we can also define some other types of the SMs based onEqs. (1), (2), (3) and (4), which are summarized in Definitions 9 and 10

Definition 9: For two SFSs P and Q, the weighted average SM of the functionsS_t^a,S_t^b, S_t^c and S_t^d is defined as

$S_y(P,Q) = \sum_{i=1}^{n} \omega_i \left(\frac{S_t^a(P,Q) + S_t^b(P,Q) + S_t^c(P,Q) + S_t^d(P,Q)}{2}\right)$............ (6)

Where $\omega_i > 0$ is the normalized weight vector of elements of B.

Theorem 3: The measure given in Definition 9is a valid SM for SFSs.

Proof. For two SFSs P and Q and from the Theorem 1, we have

(P1) Since$0 \leq S_t^a(P,Q) + S_t^b(P,Q) + S_t^c(P,Q) + S_t^d(P,Q) \leq 1$, we have

$$0 \leq S_y(P,Q) \leq \sum_{i=1}^{n} \omega_i = 1$$

(P2) It can be easily proven, so we omit here.

(P3) As$S_t^a(P,Q) = S_t^b(P,Q) = S_t^c(P,Q) = S_t^d(P,Q) = 1$ if and only if P = Q, so by the definition of P1, we get S_y(P, Q) = 1 ifonly if P = Q.

(P4) For SFSs P, Q and R such that P ⊆ Q ⊆ R $S_t^a(P,R) \leq min\{S_t^a(P,Q), S_t^a(Q,R)\}, S_t^b(P,R) \leq min\{S_t^b(P,Q), S_t^b(Q,R)\}, S_t^c(P,R) \leq min\{S_t^c(P,Q), S_t^c(Q,R)\} \; and \; S_t^d(P,R) \leq min\{S_t^d(P,Q), S_t^d(Q,R)\}$.

So that

$$S_y(P,R) \leq S_y(P,Q) \; and \; S_y(P,R) \leq S_y(Q,R)$$

Definition 10: For two SFSs P and Q and using functions S_t^a, S_t^b, S_t^candS_t^d , a generalized weighted SM,$S_q$is defined as

$$S_q\,(P,Q) = \sum_{i=1}^{n} \omega_i \left(\frac{\sqrt[p]{\left(S_t{}^{a}(P,Q)\right)^q + \left(S_t{}^{b}(P,Q)\right)^q + \left(S_t{}^{c}(P,Q)\right)^q + \left(S_t{}^{d}(P,Q)\right)^q}}{2} \right) \ldots\ldots\ldots (7)$$

For all q = 2

Theorem 4: The function S_q given in Definition10is an SM.The proof can be obtained as similar to Theorem 3.

4. Application of q-rung orthopair spherical fuzzy set to the DMPs

In this section, we states the DMPs method on q- rung orthopair fuzzy set based on the proposed SMs under SFS environment to determine the finest alternative(s).To real view , let's assume that α=$\{\alpha_1, \alpha_2, \ldots \ldots, \alpha_m\}$ be the set of "m" alternatives and β=$\{\beta_1, \beta_2, \ldots \ldots, \beta_n\}$ be the set of "n" criteria, whose weight vector is $\omega_\gamma > 0$ with $\sum_{\gamma=1}^{n} \omega_\gamma$=1. An expert evaluates these alternatives and rates then in terms of SFSs $\Omega_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ such that $0 \leq {a_{ij}}^2 + {b_{ij}}^2 + {c_{ij}}^2 + {d_{ij}}^2 \leq 1$ satisfied. The complete SF decision matrix X is defined as

$$X = \begin{pmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \cdots & \Omega_{1n} \\ \Omega_{21} & \Omega_{22} & \Omega_{23} & \ldots & \Omega_{2n} \\ . & . & . & & . \\ . & . & . & \ddots & . \\ . & . & . & & . \\ \Omega_{m1} & \Omega_{m2} & \Omega_{m3} & \ldots. & \Omega_{mn} \end{pmatrix}$$

Thus, the following steps are proposed based on the proposed q- rung orthopair spherical fuzzy set SMs to evaluate them.

Step1. In the first step we find weight of each criterion with weight vector $\omega_{j,k}$(k=0, 1, 2…) of each criterion β_j using the following equation:

$$\omega_{j,k} = \frac{(d_j)^k}{\sum_{j=1}^{n} (d_j)^k}, k = 0, 1, 2 \ldots \quad \ldots\ldots\ldots (8)$$

Where

$$d_j = d_{1j} + d_{2j} + d_{3j} + d_{4j}$$

In which $d_{1j} = maxi\ \ a_{ij}$, $d_{2j} = mini\ \ b_{ij}$, $d_{3j} = maxi\ c_{ij}\ and\ d_{2j} = mini\ d_{ij}$for all j=1, 2…n, such that$\sum_{j=1}^{n} \omega_{j,k} = 1$ for $k = 0, 1, 2 \ldots\ldots$

Step 2. To determine the ideal values

For these given criteria are divided into two disjoint sets, namely, the cost F1 and the benefit F2. For F1 criteria, the ideal values are taken as (0, 1), while for F2 criteria, we take (1, 0). It is noted here that (1, 0) is the largest value of a SFSs and (0, 1) isthe smallest value of a SFSs. Therefore, we represent the ideal values for all criteria as$\alpha_b = (\alpha_b\ (1), \alpha_b(2) \ldots \alpha_b\ (n))$, where $\alpha_b(j) = (1, 0)$ if β j∈ F2and $\alpha_b(j) = (0, 1)$ if βj ∈F1 for all $j = 1, 2 \ldots n$.

Step 3: Calculate the SMs of each alternative from its ideal values

Using the proposed SMs, i.e., S_x, S_y and S_q , compute the measurement values of each alternative. Based on the rating values and the ideal measures, compute the SMs values using S_x, S_y or S_q measures.

Step 4: Rank the alternatives

Based on the assessment values of the SMs, rank the given alternatives with the following rules:

$\alpha_i < \alpha_p$ if and only if $S\ (\alpha_i, \alpha_b)\ \leq\ S(\alpha_p, \alpha_b)$ for all $i, p\ =\ 1, 2 \ldots m$.

Here, S represents the SM.

Example 1:

To demonstrate the applicability of the proposed method, we have adapted a decision-making problem from Farid et al. [47]. Specifically, we consider the data of their work, which addresses a food waste treatment technology (FWTT) selection problem under a T-spherical fuzzy environment. The data is re-evaluated using our q-rung orthopair spherical fuzzy SM for DMP. After a preparatory investigation, a group of specialists (E) has chosen five technologies as the candidate of FWTT, which are anaerobic absorption, incineration, composting, landfill, and gasification. DMs evaluating the five technologies under the criterion given below,

Criterion for land selection

- A_1: Environmental capacity involvement with the developed scenario
- A_2: Time and space are required for residential waste management
- A_3: Number of eco-friendly purchases
- A_4: Number of participants in promoting the model

The decision matrix formed alternative and the given attributes (FWTT) is givenby:

$$E = \begin{bmatrix} (0.475,0.145,0.110)(0.345,0.252,0.130)(0.555,0.303,0.115)(0.425,0.135,0.111) \\ (0.335,0.245,0.145)(0.145,0.225,0.335)(0.230,0.153,0.420)(0.235,0.435,0.245) \\ (0.370,0.265,0.275)(0.135,0.210,0.545)(0.235,0.152,0.220)(0.265,0.225,0.433) \\ (0.220,0.465,0.135)(0.250,0.415,0.245)(0.415,0.152,0.140)(0.265,0.445,0.135) \\ (0.143,0.154,0.465)(0.270,0.458,0.165)(0.120,0.415,0.260)\ \ (0.215,0.115,0.435) \end{bmatrix}$$

By using the equation (8) we able to find the value weightage of each criteria we can get

$$\omega = \ (0.239746942, 0.251720507, 0.269844683, and\ 0.238687868)$$

As follow the step 2 we get that $\beta_4 \in$ F1, while others are belongs to F_2, So ideal value are $\alpha_b(1) = \alpha_b(2) = \alpha_b(3) = (1,0)\ and\ \alpha_b(4) = (0,1)$.

Table 1

Weighted Similarity Value of $S_x, S_y and S_q$

Alternative's	(P,Q)S_x	(P,Q)S_y	(P,Q)S_q
S_1 (P,Q)	0.01521637	0.265285907	0.144474617
S_2 (P,Q)	0.013943986	0.262540525	0.145556018
S_3 (P,Q)	0.017822146	0.293267959	0.158056111
S_4 (P,Q)	0.072511247	0.37958448	0.198985615

Finally, we utilize the similarity measure S_q to compute the evaluation values.

Table 2
Similarity Measure S_q

Alternative's	(P,Q)S_q	Ranking
A_1	**0.647072362**	**3**
A_2	**0.651155207**	**2**
A_3	**0.656005710**	**1**
A_4	**0.627382035**	**4**
A_5	**0.61633325**:	**5**

Form Table 2, we arrive at the result that FWTT A3 is the most desirable. We validate this conclusion by applying various decisions- making problem for similarity measure as illustrated in Table 3.

Table 3
Comparison Analysis and Ranking Order

Procedure	Ranking result
Method by PFC1 proposed in [26]	$A_3 > A_2 > A_5 > A_1 > A_4$
Method by PFC2 proposed in [26]	$A_3 > A_2 > A_5 > A_4 > A_1$
Method by S_m proposed in [25]	$A_3 > A_2 > A_5 > A_1 > A_4$
Method by S0 proposed in [48]	$A_3 > A_2 > A_1 > A_5 > A_4$
Method by S1 proposed in [48]	$A_3 > A_2 > A_1 > A_5 > A_4$
Method by S2 proposed in [48]	$A_3 > A_2 > A_1 > A_5 > A_4$
Method by $S_x$proposed in the paper	$A_3 > A_2 > A_4 > A_1 > A_5$
Method by S_y proposed in the paper	$A_3 > A_2 > A_4 > A_1 > A_5$
Method by S_q proposed in the paper	$A_3 > A_2 > A_1 > A_4 > A_5$

We observe that all the above methods given in Table 3 show that A_3 is the best alternatives among all other alternatives which are same as obtained by our suggested method. This proves the authenticity of our result with the concept of q-rung orthopair similarity measure method for spherical fuzzy set.

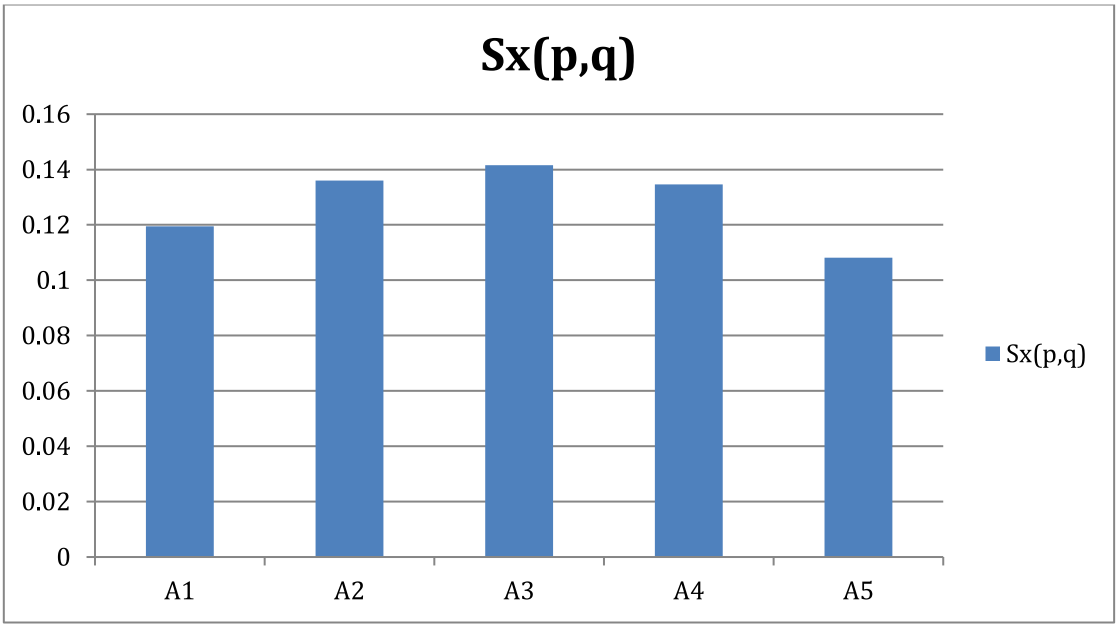

Fig.1. Graphical representation of the weighted similarity measure for FWTT alternatives

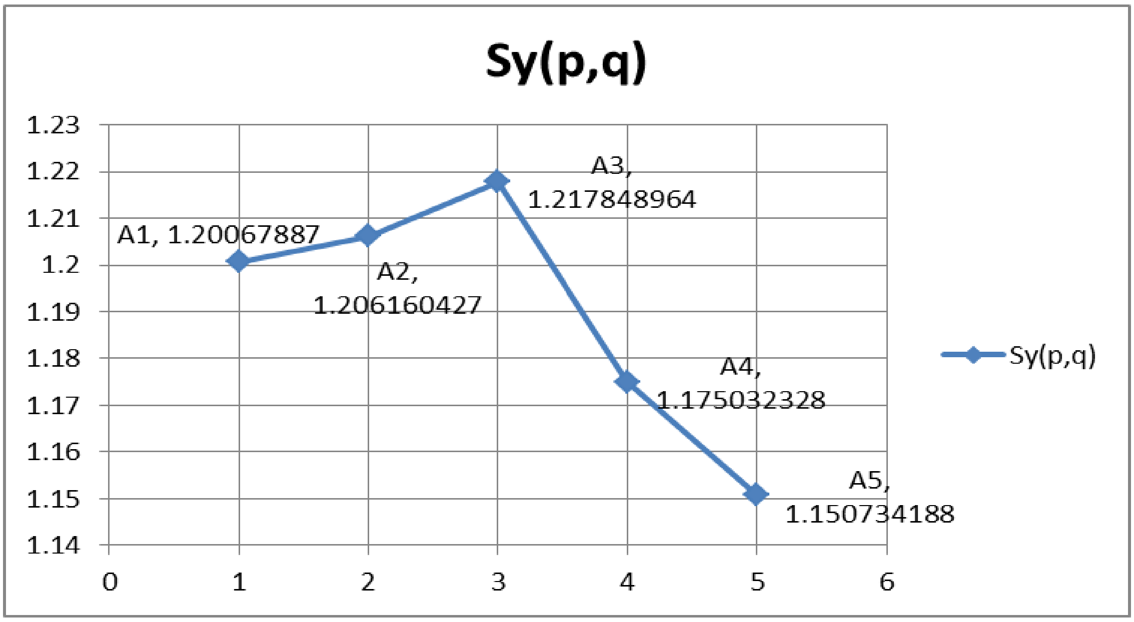

Fig.2. Graphical representation of the weighted average similarity measure for FWTT alternatives

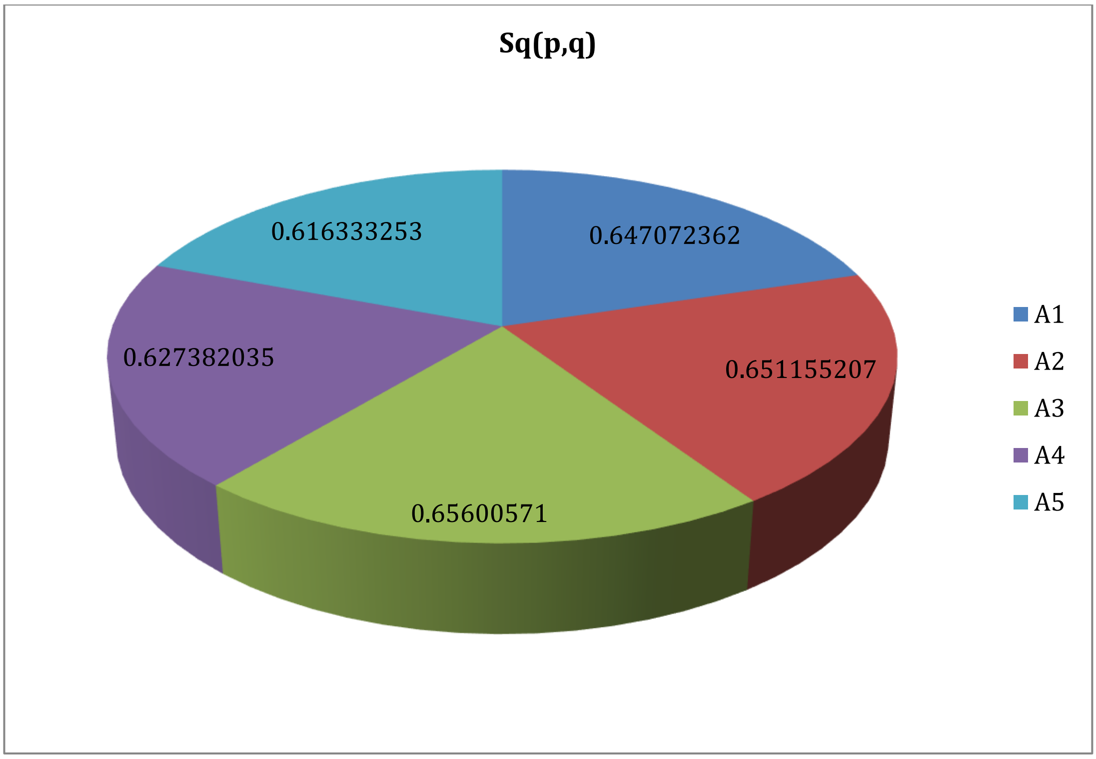

Fig.3. Graphical representation of the generalized weighted average similarity measure for FWTT alternatives

To provide a clear visual understanding of the performance of the considered FWTT alternatives, a graphical representation of the ranking scores S_x (P, Q),S_y (P, Q), S_q (P, Q) were computed for values of q=2 in order to investigate the effect of the q-rung parameter on the ranking stability of FWTT alternatives. Figure 3 provides a graphic representation of the findings. The robustness of Alternative A3 as the best food waste treatment technology is demonstrated by the fact that it continuously maintains the highest-Ranking score across a range of q=2 values. The overall ranking pattern stays constant while other options exhibit slight variations, confirming the validity of the suggested q-rung orthopair Spherical fuzzy set for decision-making problem.

Example 2: Medical Waste Management (MWM) Technology Selection

Problem Statement:

To further demonstrate the applicability of the proposed q-rung orthopair spherical fuzzy similarity measures, we consider a medical waste management (MWM) technology selection problem adapted from Sarwar et al. [63].

Let $W = \{W_1, W_2, W_3, W_4, W_5\}$ be the set of five alternative technologies, where, W_1 = Steam sterilization; W_2 = Incineration; W_3 = Chemical disinfection; W_4 = Microwave; W_5 = Landfill disposal

Let £= {$£_1$,$£_2$, $£_3$,$£_4$, $£_5$,$£_6$, $£_7$,$£_8$} be the set of eight criteria, where, $£_1$ = cost; $£_2$ = waste residuals; $£_3$ = Health impact; $£_4$ = Energy Consumption; $£_5$ = Reliability; $£_6$ = Volume reduction; $£_7$ = Treatment effectiveness; $£_8$ = Public acceptance.

The decision matrix E is given as follows (each entry is a q-RO-SFN of the form (a, b, c, d) representing satisfaction, non-satisfaction, and neutral degrees respectively):

$$E = \begin{bmatrix} (0.0180, 0.0072, 0.4490) & (0.0195, 0.0082, 0.4436) & (0.0213, 0.0080, 0.4394) & (0.0197, 0.0075, 0.4440) & (0.0192, 0.0077, 0.4455) \\ (0.0331, 0.0101, 0.4089) & (0.0318, 0.0097, 0.4129) & (0.0305, 0.0132, 0.4140) & (0.0275, 0.0098, 0.4223) & (0.0277, 0.0089, 0.4204) \\ (0.0184, 0.0091, 0.4455) & (0.0145, 0.0091, 0.4542) & (0.0224, 0.0100, 0.4358) & (0.0194, 0.0115, 0.4428) & (0.0199, 0.0089, 0.4410) \\ (0.0342, 0.0127, 0.4068) & (0.0325, 0.0094, 0.4090) & (0.0267, 0.0127, 0.4229) & (0.0244, 0.0136, 0.4285) & (0.0308, 0.0135, 0.4165) \\ (0.0237, 0.0075, 0.4334) & (0.0254, 0.0093, 0.4293) & (0.0241, 0.0106, 0.4331) & (0.0229, 0.0093, 0.4369) & (0.0205, 0.0119, 0.4399) \\ (0.0165, 0.0078, 0.4519) & (0.0166, 0.0079, 0.4516) & (0.0174, 0.0080, 0.4493) & (0.0149, 0.0078, 0.4553) & (0.0212, 0.0078, 0.4388) \\ (0.0219, 0.0089, 0.4366) & (0.0215, 0.0064, 0.4398) & (0.0228, 0.0110, 0.4358) & (0.0262, 0.0099, 0.4271) & (0.0239, 0.0082, 0.4321) \\ (0.0318, 0.0140, 0.4106) & (0.0196, 0.0114, 0.4417) & (0.0304, 0.0131, 0.4150) & (0.0196, 0.0089, 0.4390) & (0.0244, 0.0118, 0.4297) \end{bmatrix}$$

By using the equation (8) we able to find the value weightage of each criterion we can get

$$\omega = (0.097805711, 0.109074761, 0.128583741, 0.150933417, 0.128782476, 0.128597375, 0.128553465 \text{ and} 0.127669055)$$

The criteria types are as follows:

$£_1$, $£_2$, $£_3$, $£_4$ are cost criteria (lower values are better), while $£_5$, $£_6$, $£_7$, $£_8$ are benefit criteria (higher values are better)

Table 1

Weighted Similarity Value of $S_x, S_y and S_q$

Alternative's	(P,Q)S_x	(P,Q)S_y	(P,Q)S_q
W_1	**0.114456366**	**0.643292281**	**0.350517941**
W_2	**0.114039848**	**0.642453223**	**0.349998541**
W_3	**0.114396018**	**0.643173398**	**0.350441518**
W_4	**0.113893318**	**0.642154483**	**0.349810979**
W_5	**0.114238737**	**0.642859665**	**0.350247458**

Table 5

Comparison analysis and ranking order

Method	W_1	W_2	W_3	W_4	W_5	Best ranking
Proposed S_x(P,Q)	**0.114456366**	**0.11403984**	**0.11439601**	**0.11389331**	**0.11423873**	W_1
Proposed S_y(P,Q)	**0.643292281**	**0.64245322**	**0.64317339**	**0.64215448**	**0.64285966**	W_1
Proposed S_q(P,Q)	**0.350517941**	**0.34999854**	**0.35044151**	**0.34981097**	**0.35024745**	W_1
SF-TOPSIS [58]	**0.65406**	**0.46501**	**0.52051**	**0.26560**	**0.52356**	W_1
IF-TOPSIS [60]	**0.67447**	**0.42379**	**0.57421**	**0.22445**	**0.53974**	W_1
F-TOPSIS [59]	**0.72264**	**0.31361**	**0.71694**	**0.26336**	**0.72020**	W_1
SF-WASPAS [61]	**0.65690**	**0.65620**	**0.65810**	**0.65490**	**0.65610**	W_3
IF-WASPAS [62]	**0.76470**	**0.73560**	**0.76280**	**0.61630**	**0.73960**	W_1

We observe that all the above methods given in Table 5 show that W_1 is the best alternatives among all other alternatives which are same as obtained by our suggested method. This proves the authenticity of our result with the concept of q-rung orthopair similarity measure method for spherical fuzzy set.

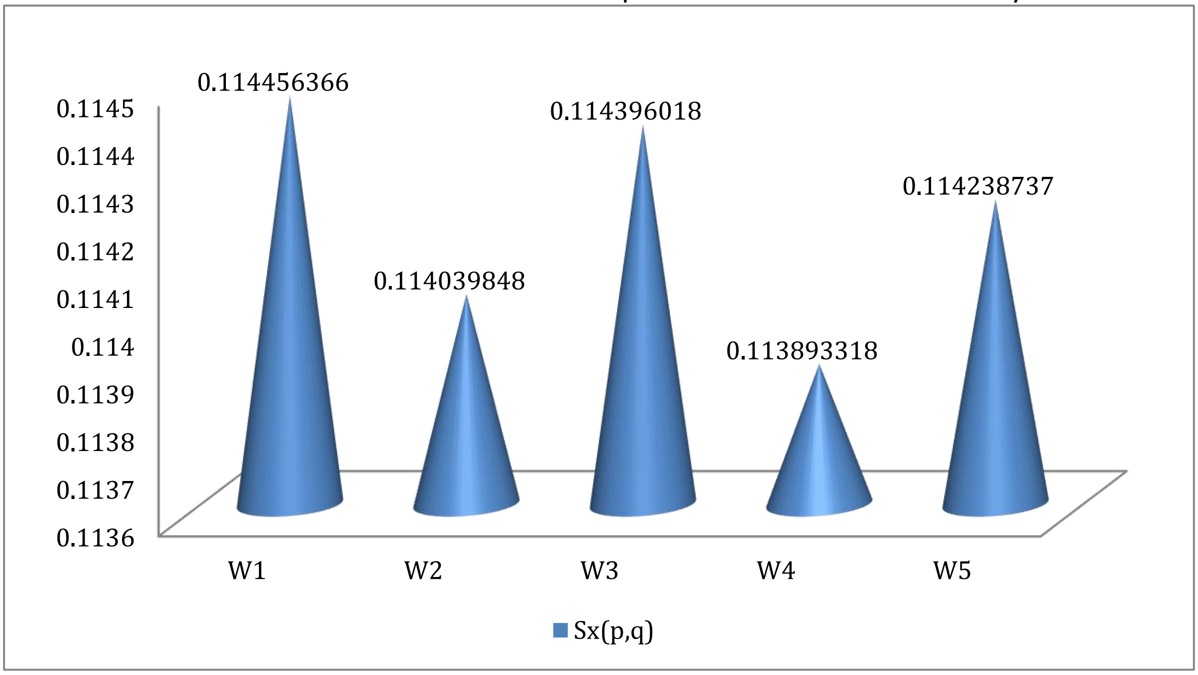

Fig.4. Graphical representation of the weighted similarity measure for MWM alternatives

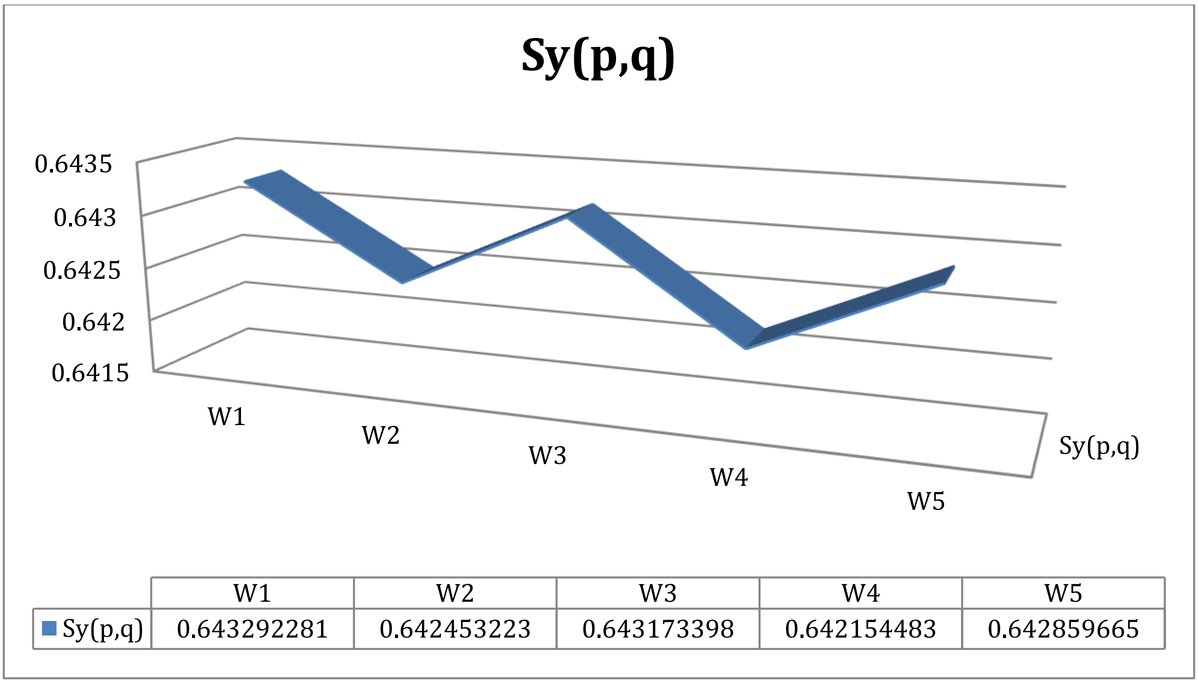

Fig.5 Graphical representation of the weighted average similarity measure for MWM alternatives

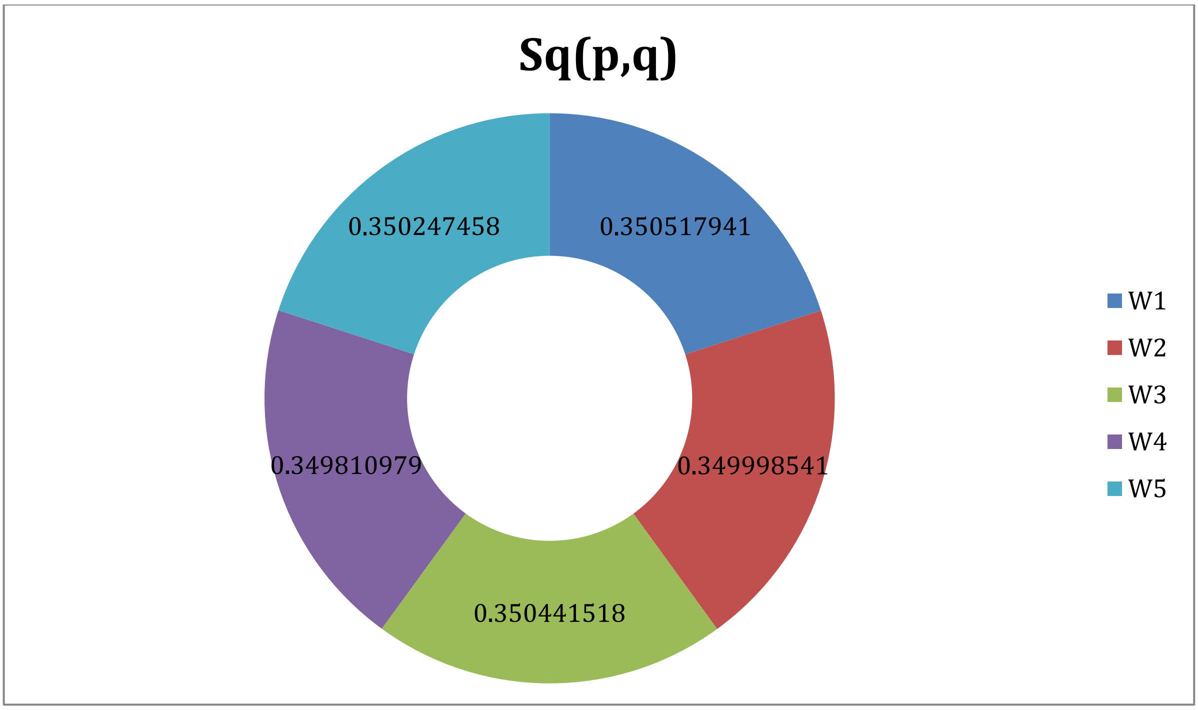

Fig.6 Graphical representation of the generalized weighted average similarity measure for MWM alternatives

From the computed similarity values, it is observed that Alternative W1 attains the highest ranking among all the considered medical waste management technologies. This indicates that W1 is the most appropriate option based on the selected criteria, as it demonstrates a higher degree of satisfaction and comparatively lowers uncertainty components. This result reflects that W1 performs better in terms of critical factors such as cost efficiency, reliability, treatment effectiveness, and public acceptance. The proposed similarity measures successfully capture the trade-offs among different criteria and provide a clear ranking of alternatives. Although other alternatives exhibit competitive performance, their slightly lower similarity values indicate marginally less suitability. Overall, the results confirm that the proposed q-rung orthopair spherical fuzzy framework is capable of handling complex and uncertain decision-making problems in healthcare applications, providing stable and meaningful rankings.

5. Sensitivity Analysis

To evaluate the robustness of the proposed q-rung orthopair spherical fuzzy similarity measures, a sensitivity analysis is performed for the considered case studies. In this analysis, slight variations in the criteria weights are introduced to examine the influence of decision-makers' preferences on the ranking results. The obtained results show that the ranking order of the top alternatives remains unchanged, while only minor variations are observed in the similarity values of other alternatives. These observations indicate that the proposed method is stable and reliable under small perturbations in input data. The framework effectively handles uncertainty and provides consistent decision-making outcomes.

6. Advantages and Limitations of the Proposed Method

In this section, the key advantages and limitations of the proposed q-rung orthopair spherical fuzzy similarity measures are discussed to provide a comprehensive understanding of their applicability in real-world decision-making problems.

Advantages:

The proposed method offers several significant advantages. First, it incorporates four components, namely satisfaction, non-satisfaction, neutrality, and hesitancy, which enables a more comprehensive representation of uncertainty compared to traditional fuzzy models such as intuitionistic and picture fuzzy sets. Second, the inclusion of the q-rung parameter provides additional flexibility, allowing decision-makers to model varying degrees of uncertainty more effectively. Third, the use of exponential similarity measures enhances the sensitivity and discrimination capability of the model, leading to more precise ranking results. Furthermore, the proposed framework is capable of handling multi-criteria decision-making problems in diverse application areas such as environmental management and healthcare. The results obtained in the case studies demonstrate that the method provides stable and consistent rankings, even in the presence of uncertainty and slight variations in input data.

Limitations:

Despite its advantages, the proposed method has certain limitations. The selection of the q-rung parameter plays a crucial role in the model, and inappropriate choice of q may affect the final results. Additionally, the method relies on expert judgment for constructing the decision matrix, which may introduce subjectivity into the decision-making process. The computational complexity may also increase as the number of criteria and alternatives grows, particularly when dealing with large-scale problems. Moreover, the current study focuses on static decision-making scenarios and does not consider dynamic or time-dependent environments. Future research may extend the proposed approach to group decision-making, dynamic systems, and hybrid models by integrating entropy-based or distance-based techniques.

7. Conclusion:

In this paper, a novel framework for exponential similarity measures under the q-rung orthopair spherical fuzzy environment has been proposed. By extending traditional spherical fuzzy sets to incorporate a q-rung orthopair structure with four components, satisfaction, non-satisfaction, neutrality, and hesitancy, the proposed approach provides a more comprehensive representation of uncertainty in decision-making problems. The developed similarity measures satisfy important mathematical properties such as boundedness, symmetry, identity, and monotonicity. In addition, weighted and generalized models have been introduced to accommodate varying importance of criteria in practical multi-criteria decision-making scenarios. The applicability and effectiveness of the proposed method have been demonstrated through two case studies, namely food waste treatment technology selection and medical waste management.

The results indicate that the proposed similarity measures produce stable and reliable rankings and offer improved discrimination capability compared to existing methods.

The main advantages of the proposed approach include its flexibility in handling multiple uncertainty components and enhanced sensitivity achieved through exponential functions. However, certain limitations exist, such as dependence on the selection of the q-rung parameter and the involvement of expert judgment in constructing the decision matrix, which may introduce subjectivity.

Overall, the proposed methodology enhances the expressive power of fuzzy decision-making models, particularly in environments involving ambiguity, hesitation, and neutrality. Future research can extend this work to group decision-making, dynamic environments, and large-scale problems. Moreover, hybrid approaches integrating entropy-based and distance-based techniques can be explored to further improve decision-making performance.

Author Contributions

Conceptualization, Antaksh Prashar and Abhishek Guleria; methodology Preeti Devi, Anil Kumar; writing—original draft preparation, Antaksh Prashar, Neetu Dhiman, Mamta Devi; review and editing Arun Kumar, Abhishek Guleria; supervision Abhishek Guleria.

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Data Availability Statement
In this section, please provide details regarding where data supporting reported results can be found, including links to publicly archived datasets analyzed or generated during the study. You might choose to exclude this statement if the study did not report any data.

Conflicts of Interest
Declare conflicts of interest or state "The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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