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Bridging Art and Logic: Evolving Artificial Intelligence for Emotionally Intelligent Art Communication Using Integrated PROMETHEE – Complex Circular Intuitionistic Fuzzy Dombi Framework

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ABSTRACT

In modern communication systems, especially in art communication, emotional intelligence is essential in improving engagement and sympathy between audiences and creators. Existing decision-making models fail when faced with the complexity and subjectivity of human emotions, particularly in situations marked by vagueness and cultural diversity. This study aims to introduce the Dombi operational law model based on complex circular intuitionistic fuzzy (C-CrIF) information. The prioritized aggregation operators (AOs) involve assigning multiple levels of significance to the fuzzified criteria, the alternatives, and the expert opinions before combining them. We proposed new AOs, Dombi operational laws under the C-CrIF set (C-CrIFS), including complex circular intuitionistic fuzzy Dombi prioritized weighted averaging and complex circular intuitionistic fuzzy Dombi prioritized weighted geometric operators. These AOs are the improved forms of Dombi and prioritized AOs for fuzzy, complex fuzzy, intuitionistic fuzzy, complex intuitionistic fuzzy, and complex circular intuitionistic fuzzy data. Some essential properties of the proposed operators are also proposed. Also, we proposed the preference ranking organization method for enrichment evaluation (PROMETHEE) method based on the C-CrIF framework. Next, to demonstrate the applicability of established AOs, a case study of emotionally intelligent art communication is conducted in the presence of the proposed operators and using the PROMETHEE method. By combining these methods, the model can identify the most emotionally resonant communication alternatives, offering artists, curators, and artificial intelligence agents a powerful tool to improve audience connection. Lastly, by comparing the existing models, we provide clarifications on the established model to demonstrate the superiority and value of the developed model.

1. Introduction

Art communication conveys the dynamic arguments of emotions, ideas, and cultural values through various creative media, fostering human expression and audience understanding. Emotional

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intelligence must be leveraged to interpret and respond to subtle psychological and sensory cues between creators and audiences, fostering meaningful, lasting connections, especially in immersive or digital art contexts. The complexity of modern art interactions, amplified by digital transformation, cross-cultural diversity, and rapidly evolving audience preferences, requires plans that address subjective experiences and technological borders. In recent years, during COVID-19, the art world has faced deep challenges in maintaining engagement, authenticity, and emotional connection amid social distancing and virtual migration. The long-term goal in this new normal is not to preserve artistic experiences but to improve them through emotionally intelligent systems that can adapt, personalize, and respond in real time. Art communication today is no longer just a creative effort. It is a complex, multi-dimensional interaction that calls for advanced decision-making contexts to model human feelings, vagueness, and emotional resonance. So, finding the most effective emotional engagement approaches becomes a typical multi-attribute decision-making (MADM) challenge, requiring a hybrid approach that integrates psychology, technology, and fuzzy judgment to optimize human-centered communication.

Many valuable models, such as decision-making procedures and clustering analysis, are applied to assess the best option based on multiple attributes. The MADM model is the best decision-making technique and is also used to select better options from numerous alternatives under the classical set (CS) theory. However, CS has two possibilities, as 0 or 1. However, in many real-life situations, we require more possibilities. So, to address this issue, Zadeh [1] established the theory of fuzzy sets (FSs). FS is advanced as compared to CS because the range of FS is extensive and also described in detail from $[0, 1]$. Many applications and generalizations of FS have been established, such as Kharaman [2] established FS information and application in industrial engineering, Samimi Namin and Rouhani [3] established FS theory for Rock Engineering, Valdes [4] proposed bibliometric review of type-3 Fuzzy Logic Applications and Hazaymeh [5] established application in design-making with Time Fuzzy Soft Sets. Saad et al. [6] used fuzzy-AHP model for procurement process, Josen et al. [7] established a multidimensional FS based on distance, similarity, and entropy, and Mageed [8] established fractal FS for future outlooks and open conjectures. The solution of advanced MADM problems using the Muirhead mean AOs provided by Hussain et al. [9] and the concept of the MADM problem using the Aczel-Alsina operations given by Hussain et al. [10].

The FS theory is highly reliable and well-known because it has attracted significant attention from many researchers. However, FS theory only expresses a satisfactory degree; it cannot express hesitation or a dissatisfactory degree in decision-making. In many frameworks, hesitation or dissatisfaction is a feature of real-life problems. To overcome this situation, Atanassov [11] established the intuitionistic fuzzy set (IFS) theory, which defined the satisfactory and dissatisfactory degree, allowing a hesitancy degree to take indecision or uncertainty information, which improves in multi-criteria evaluations. The IFS range is extensive compared to FS theory, such as the sum of satisfactory and dissatisfactory degrees, which will be covered in the unit interval. Many applications and various generalizations of the IFS have been established by researchers, such as Ortiz-Barrios et al. [12] proposed IF numbers and an application for disaster preparedness in clinical laboratories, Uluçay and Şahin [13] established IF soft expert graphs and their application, Xie et al. [14] proposed an application in decision making based on IFS, Görçün et al. [15] given the idea of an IF consensus-based method and assessment of crawler cranes for large-scale construction and infrastructure developments.

However, we occasionally fail to address the real-world problem of periodicity in two-dimensional data. FS only handles one-dimensional data at a time. To overcome this issue, Ramot et al. [16] proposed the complex fuzzy set (C-FS) theory, which defined periodic terms within a satisfactory degree framework and provided a novel form of complex value. Many scholars used C-FS in numerous applications and generalizations of C-FS, such as Saif et al. [17], who proposed AI-driven predictive maintenance using an improved TOPSIS method for C-FS with Z-numbers, Rehman et al. [18] established Bipolar C-FS Muirhead Mean decision-making method for Social Issues Strategies, Qiyas et al. [19] proposed a decision-making application based on Confidence levels bipolar C-FS AOs, and Mahmood et al. [20] invented an application of decision-making under Bipolar complex fuzzy soft sets. Still, the C-FS has only discussed satisfaction, not dissatisfaction. Because a dissatisfactory degree plays a vital role in the decision-making process, Alkouri and Salleh [21] introduced the concept of the complex intuitionistic fuzzy set (C-IFS), which defines amplitude and phase terms for satisfactory and dissatisfactory degrees using a practical, widely accepted possibility measure. The C-IFS is an advanced form of C-FS theory.

Many other studies used C-IFS across numerous applications and frameworks, such as Ali et al. [22] invented the model of C-IFS under Archimedean Heronian mean operators and decision-making applications, Liu and Ali [23] established applications in green supply chain management based on Hamacher interaction aggregation operators with C-IFS, Alghazzawi et al. [24] established C-IF dynamic environment for selecting the optimal method for cardiovascular disease diagnosis and Azam [25] invented C-IFS for the selection of a computer network security system based on dual partitioned Maclaurin symmetric mean operators. However, hesitation or sensitivity cannot be combined directly. This gap is associated with the circular intuitionistic fuzzy set (CrIFS) invented by Cakir and Tas [20], which defines satisfactory and dissatisfactory as vectors on a circular surface. Chen [26] used CrIFS for multiple criteria optimization and compromise solutions, and Unni et al. [27] established the CrIF soft-set-theoretic method for decision-making complications. Then, the Complex Circular Intuitionistic Fuzzy Set (C-CrIFS) combines the intuitionistic framework with complex-valued illustrations. In C-CrIFS, satisfactory and dissatisfactory are denoted as vectors on a complex circular surface, capturing the strength and direction of belief alongside a hesitation degree. Kong [28] proposed C-CrIF Heronian mean AOs for dynamic air quality monitoring and public health risk prediction. C-CrIFS allows the system to model how strongly a decision leans toward acceptance or rejection, as well as its angular orientation, which is ideal for handling opposing views, conflicting emotions, or dynamic connections. C-CrIFS is particularly powerful in domains such as emotion-aware decision systems, interactive AI, and art communication, where data is uncertain, multidimensional, and subjective. C-CrIFS offers the most advanced structure for modeling ambiguous, emotionally stimulating, and direction-sensitive decisions, marking a significant leap in fuzzy set theory. Also, we compare the C-CrIFS with another fuzzy extension in Table 1.

Table 1

Comparison of fuzzy extension with C-CrIFS

Feature	Fuzzy Set (FS)	Intuitionistic Fuzzy Set (IFS)	Neutrosophic Set (NS)	Complex Fuzzy Set (C-FS)	Complex Circular Intuitionistic Fuzzy Set (C-CrIFS)
Satisfactory degree	$[0, 1]$	$[0, 1]$ for membership & non-membership	$[0, 1]$ for truth, indeterminacy, falsity	Complex value	Complex value + IFS components
Dissatisfactory degree	depend on a satisfactory degree	Explicit	Explicit and independent	Derived from a complex function	Explicit, complex-valued
Hesitancy degree $Л$	Not Defined	defined	$Л =$ independent (any [0,1])	Not handled explicitly	Clearly defined and complex
Handles Ambiguity / Uncertainty	Limited	Better handling	Very high	Moderate	Very high
Cam model opposing opinions (Phase)	No	No	Yes	Yes	Yes
Application complexity	Low	Medium	High	Medium	High
Best use cases	Basic decision systems	Multi-criteria decision making	Complex logical systems	Signal processing, control	Emotion-aware, uncertain, dynamic decision systems

1.1 Aggregation operators

The modification of AOs within this fuzzy framework has been ongoing work; in 1982, Dombi [4] introduced the Dombi t-norm (DTNM) and the Dombi t-conorm (DTCNM), which are well known for their significant variability relative to traditional triangular norms. The AOs play a significant role in assembling C-CrIF information, and the modifiable parameters in DTNM and DTCNM operations offer them unparalleled flexibility, making them particularly suited for developing AOs. Many researchers used DTNM and DTCNM in numerous fields, such as Ali and Mahmood [29], who proposed the MADM problem based on the Dombi aggregation operator; Jalal [30] proposed Dombi AOs based on complex fuzzy soft information; Jana [31] established a bipolar fuzzy set based on Dombi DTNM and DTCNM, and the application of MADM, Khan et al. [32] invented IF Dombi AOs under lower and upper approximation, Riaz et al. [33] cubic bipolar fuzzy Dombi averaging AOs with application and Mehmood et al. [34] established hesitant bipolar fuzzy Dombi AOs for selecting a cloud service provider. Yang et al. [35] established multi-attribute fusion decision making based on the Aesthetic evaluation method, Zhang et al. [36] established multi-attribute group decision-making based on IFS and Hassain et al. [37] established MADM based on T-spherical fuzzy Aczel Alsina Heronian mean operators. The concept of complex AOs based on the Aczel-Alsina operation discussed by Hussain et al. [38] and the theory of complex Pythagorean FS-based AOs for the precise solution of MADM problems given by Jin et al. [39]. Concept of novel AOs based on the complex Picture FS framework under Frank operations provided by Hussain et al. [40] and the solution to the supplier selection problem using MADM under prioritized AOs, as discussed by Hussain et al. [41].

1.2 Importance of the PROMETHEE method

The PROMETHEE method plays a vital role in fuzzy models by allowing effective MADM under ambiguity. It integrates a fuzzy model to handle ambiguous, vague, and hesitant data, which are common in real-world situations. Using pairwise comparisons and preference functions, alternatives are ranked based on positive and negative flows. This technique improves decision clarity and supports complex reasoning in emotionally or subjectively motivated situations. This flexible method, combined with various fuzzy models, makes it ideal for advanced decision-support systems. Many researchers use the PROMETHEE method, such as Gul et al. [42] proposed the PROMETHEE method for material selection; Samanlioglu and Ayag [43] proposed AHP-PROMETHEE II method for evaluation of solar power plant location, Senver et al. [44] Invented the PROMETHEE Method for Supplier Selection, Liao and Xu [45] proposed the IF PROMETHEE method and [46] proposed model for public transport mode-choice analysis based on the fuzzy PROMETHEE technique.

1.3 Problem statement and literature review

In digital and interconnected worlds, art communication has extended beyond traditional limitations, involving complex interactions among audiences, artists, and intelligent systems. The developments in multimedia and virtual stages, the complexity of human emotion, and the gradation of explanation in art remain difficult to quantify and convey effectively. Current communication systems fail to identify the emotional context of artistic expression, leading to a loss of authenticity and engagement. The absence of emotional intelligence in these systems limits their capability to adapt to human sentimental situations, reducing the impact of the communication method. So, a novel framework is required that integrates emotional intelligence into art communication, enabling systems to understand, respond to, and collaborate with users. The challenge lies in structuring emotion-aware decision-making models that can handle ambiguity, subjectivity, and multi-modal real-time input to enhance human-art communication and ensure meaningful engagement. Many other researchers worked on it, such as Sinha and Sinha [47], who established Emotional intelligence and operative communication; Bird [48] proposed emotional intelligence; Rezasi [49] proposed the role of arts education in helping emotional intelligence, Peng [50] proposed Fusing Emotion and Art communication based on Interval-Valued Fermatean Fuzzy Decision-Making Method, Liu et al. [51] established personalized emotion investigation with a fuzzy multi-modal transformer model. Indraneel and Miriyala [52] invented artificial intelligence-based Emotion Recognition from Fuzzy EEG Signals. We display the role of abstract art in visual communication in Figure 1.

Fig. 1. The role of abstract art in visual communication.

1.4 Motivation and scientific contribution

In complex real-world decision-making situations, such as emotional modeling, art communication, and medical diagnosis, information is frequently ambiguous, inaccurate, and emotionally influenced. Existing fuzzy frameworks effectively handle vagueness but are limited in accounting for hesitation, directional tendencies, and interrelated emotional dimensions. So, existing MADM approaches lack the sensitivity needed to process circular relationships and emotional fluctuations in human decisions. This gap motivates the development of a more sensitive and flexible fuzzy decision-making method that can accommodate both IFS and complex-valued circular behavior. However, real-life difficulties also need the system to respect the importance and order of decision attributes, where prioritized weighted aggregation becomes vital. Also, the PROMETHEE method provides a straightforward, preference-sensitive ranking approach, making it ideal for selecting the best alternatives in a fuzzy environment. The combination of our techniques is thus motivated by the need to propose a multidimensional, emotionally aware, and mathematically rigorous decision-making approach that more accurately replicates human intelligence than existing methods.

In this paper, we proposed a novel hybrid MADM model that combined C-CrIFS with the PROMETHEE method and Dombi AOs, namely, the complex circular intuitionistic fuzzy Dombi prioritized weighted averaging (C-CrIFDPWA) and the complex circular intuitionistic fuzzy Dombi prioritized weighted geometric (C-CrIFDPWG) operators. The key scientific contributions of our work are listed below:

1. We proposed prioritized aggregation in the C-CrIF framework; this technique integrates a prioritization mechanism that respects attributes' sequence and relative importance, which is essential in emotionally weighted decisions.
2. The proposed model takes the satisfactory, dissatisfactory, hesitation, and phase degrees, allowing significant MADM in emotional, sentimental, and subjective domains.
3. The Dombi AOs offer averaging and geometric forms with prioritized weight, allocating melodious sensitivity in decision-making.

4. The PROMETHEE method perfects the fuzzy decision-making step by delivering a strong, rank-based assessment with positive and negative outranking flows suitable for situations with multiple participants and incompatible decisions.
5. Our established technique is highly adaptable and applied in many areas, such as artificial emotional intelligence, interactive digital media, medical diagnosis, supplier selection, risk assessment, education systems, and innovative environments.

1.5 Contribution and novelty of the research

In this research, we present a novel hybrid MADM framework that integrates C-CrIFS with Dombi prioritized aggregation operators and the PROMETHEE method to address uncertainty and hesitation in emotionally intelligent art communication. We established some new operators, namely the C-CrIFDPWA and C-CrIFDPWG operators. These operators effectively handle complex-valued emotional and subjective information while prioritizing attributes and expert opinions. Existing fuzzy decision-making approaches, the established framework simultaneously captures satisfaction, dissatisfaction, hesitation, and phase information within a unified environment, making it highly suitable for dynamic and emotion-aware applications. Also, combining the PROMETHEE outranking technique with C-CrIFS enhances ranking accuracy and decision reliability in complex multi-criteria environments. The practical applicability and superiority of the proposed model are validated through a real-world case study on emotionally intelligent art communication and comparative analysis with existing aggregation methods.

1.6 Distinct advantages of the proposed methodology

This established methodology is not only an assembly of existing MADM techniques. This technique established a combined, precisely improved decision-making framework that overcomes several limitations of existing approaches. In various predictable MADM models, our established framework integrates C-CrIFS with Dombi aggregation operators and the PROMETHEE outranking method to concurrently address uncertainty and hesitation in complex decision-making situations. Existing methods address only membership and non-membership information, whereas our established model also captures circular and complex-valued emotional behavior. This is essential in emotionally intelligent art communication. Furthermore, the established C-CrIFDPWA and C-CrIFDPWG operators incorporate weighting mechanisms, enabling the model to better reflect the ranked importance of attributes and expert preferences. This technique enables adaptable decision-making under varying risk attitudes. Comparative and parametric analyses also prove that our established framework produces more stable, expressive, and reliable ranking results.

1.7 Organization of proposed work

Section 2 provides a detailed study of existing theories, including C-CrIFS, score functions, accuracy functions, and the Dombi aggregation operator. Section 3 proposes some Dombi-prioritized AO laws based on C-CrIFS and AOs (C-CrIFDPWA and C-CrIFDWG), and provides a detailed study of their essential properties. In Section 4, we proposed an MADM algorithm based on C-CrIFS. Section

5 presents a practical application of our established model through a real-world case study, demonstrating the method's usefulness and efficiency in complex decision-making situations. It also discusses the PROMETHEE method based on C-CrIFS and its practical application to a real-life problem. To highlight the efficiency of our established method, Section 6 presents a comparative investigation of existing operators, highlighting their advantages and usefulness, and analyzes the parameter space and discusses the practical implementation of our proposed AOs. In the end, Section 7 presents our article's conclusion, key results, and future directions for investigation.

2. Preliminaries

This section offers some basic definitions of the proposed model. It helps to understand the ambiguous and complex information.

Definition 1. [53] Assume a non-empty collection Ə. A C-CrIFS, N is described as:

$$N = \{\langle r, Ʒ(r)e^{i2\pi ȹ(r)}, ɱ(r)e^{i2\pi Ƴ(r)}, Ċ(r)e^{i2\pi ȡ(r)}\rangle : x \in Ə\}, \quad (1)$$

Where $(Ʒ(r), ȹ(r)) : Ə \to [0,1]$ and $(ɱ(r), Ƴ(r)) : Ə \to [0,1]$ are SG and DSG, and $(Ċ(r), ȡ(r)) : Ə \to [0, \sqrt{2}]$ be the radius of the circle. Also, satisfying the state $0 \le Ʒ(r) + ɱ(r) \le 1$ and $0 \le ȹ(r) + Ƴ(r) \le 1$. $Ʌ$ shows the hesitancy value $Ʌ(r) = (1 - Ʒ(r) - ɱ(r))$ and $Ʌ^0(r) = (1 - ȹ(r) - Ƴ(r))$. For convenience the $(ɱ(r), Ƴ(r))$ is called C-CIFVs.

The score function plays a crucial role in decision-making, as it is essential for ranking multiple alternatives to make the best selection. This contributes to estimating all alternatives, refining algorithms, and assessing the performance of various options.

Definition 2. [54] Assume $N = (Ʒ(r)e^{i2\pi ȹ(r)}, ɱ(r)e^{i2\pi Ƴ(r)}, Ċ(r)e^{i2\pi ȡ(r)})$ be any C-CrIFVs. Then, the score function $\underset{\circ}{A}$ for the C-CrIFVs is described as:

$$\underset{\circ}{A}(N) = \left(\frac{1}{2}\right)\Big(\big(Ʒ(r) + ȹ(r)\big) - \big(ɱ(r) + Ƴ(r)\big)\Big) \times \big(Ċ(r) - ȡ(r)\big), \quad (2)$$

where $\underset{\circ}{A}(N) \in [-1, 1]$.

The accuracy function helps verify the correctness and reliability of outcomes, particularly in contexts involving fuzzy judgment or decision-making. It also determines how close the outcome is to the actual or predictable value. It enhances solutions in machine learning and data analysis.

Definition 3. [54] Assume $N = (Ʒ(r)e^{i2\pi ȹ(r)}, ɱ(r)e^{i2\pi Ƴ(r)}, Ċ(r)e^{i2\pi ȡ(r)})$ be any C-CrIFVs. So, the accuracy function ß for the C-CrIFVs is explicated as:

$$ß(N) = \left(\tfrac{1}{2}\right)\Big(\big(Ʒ(r) + ȹ(r)\big) + \big(ɱ(r) + Ƴ(r)\big)\Big) \times \big(Ċ(r) + ȡ(r)\big), \quad (3)$$

where $ß(N) \in [0, 2]$.

If $N_a = (Ʒ_a(r)e^{i2\pi ȹ_a(r)}, ɱ_a(r)e^{i2\pi Ƴ_a(r)}, Ċ_a(r)e^{i2\pi ȡ_a(r)})$ and $N_z = (Ʒ_z(r)e^{i2\pi ȹ_z(r)}, ɱ_z(r)e^{i2\pi Ƴ_z(r)}, Ċ_z(r)e^{i2\pi ȡ_z(r)})$ be two C-CrIFVs, So,

1. If $\underset{\circ}{A}(N_a) > \underset{\circ}{A}(N_z)$, then $N_a > N_z$,
2. If $\underset{\circ}{A}(N_a) < \underset{\circ}{A}(N_z)$, then $N_a < N_z$,
3. If $\underset{\circ}{A}(N_a) = \underset{\circ}{A}(N_z)$, then
   - If $ß(N_a) > ß(N_z)$, then $N_a > N_z$,

- If $ß(N_a) < ß(N_z)$**,** then $N_a < N_z$,
- If $ß(N_a) = ß(N_z)$**,** then $N_a = N_z$.

T-norms and T-conorms are critical mathematical operations in fuzzy judgment, used to model collected information. They play a critical role in situations where objects are neither strictly accurate nor strictly untrue, but rather fall somewhere in between. These instruments support handling ambiguity across domains such as decision-making, AI, and machine learning.

Definition 4: [55] Let ȿ and ƫ be two fundamental values. So, Dombi TNM and TCNM are expressed by:

$$Dom(\text{ȿ},\text{ƫ}) = \frac{1}{1+\left\{\left(\frac{1-\text{ȿ}}{\text{ȿ}}\right)^{\mathrm{g}} + \left(\frac{1-\text{ƫ}}{\text{ƫ}}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}} \tag{4}$$

$$Dom^{0}(\text{ȿ},\text{ƫ}) = 1-\frac{1}{1+\left\{\left(\frac{\text{ȿ}}{1-\text{ȿ}}\right)^{\mathrm{g}} + \left(\frac{\text{ƫ}}{1-\text{ƫ}}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}} \tag{5}$$

Where $\mathrm{g} \geq 1$ and $(\text{ȿ},\text{ƫ}) \in [0,1] \times [0,1]$.

3. DOMBI PRIORITIZED AGGREGATION OPERATORS FOR C-CRIF DATA

Before developing the Dombi prioritized AOs, we must explain the essential Dombi operational laws within the C-CrIFVs framework. Next, to utilize the Dombi operational laws for C-CrIFVs, we propose some AOs, such as C-CrIFDPWA and C-CrIFDPWG operators. Also, we must study the established operator's essential properties, such as idempotency, monotonicity, and boundedness.

Definition 5: Let $N_a = \left(\text{Ʒ}_a(r)e^{i2\pi\varphi_a(r)}, \text{ɱ}_a(r)e^{i2\pi\text{ƴ}_a(r)}, \dot{C}_a(r)e^{i2\pi\text{ɖ}_a(r)}\right)$ and $N_z = \left(\text{Ʒ}_z(r)e^{i2\pi\varphi_z(r)}, \text{ɱ}_z(r)e^{i2\pi\text{ƴ}_z(r)}, \dot{C}_z(r)e^{i2\pi\text{ɖ}_z(r)}\right)$ be C-CrIFVs, and $\mathrm{g} \geq 1$ and $\xi > 0$ be the real value. So, Dombi TNM and TCNM operations of C-CrIFVs are expressed by:

1. $$N_a \oplus N_z = \begin{pmatrix} 1-\frac{1}{1+\left\{\left(\frac{\text{Ʒ}_{\breve{\alpha}}}{1-\text{Ʒ}_{\breve{\alpha}}}\right)^{\mathrm{g}}+\left(\frac{\text{Ʒ}_{\beta}}{1-\text{Ʒ}_{\beta}}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}}\cdot e^{i2\pi\left(1-\frac{1}{1+\left\{\left(\frac{\varphi_{\breve{\alpha}}}{1-\varphi_{\breve{\alpha}}}\right)^{\mathrm{g}}+\left(\frac{\varphi_{\beta}}{1-\varphi_{\beta}}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}}\right)}, \\ \frac{1}{1+\left\{\left(\frac{1-\text{ɱ}_{\breve{\alpha}}}{\text{ɱ}_{\breve{\alpha}}}\right)^{\mathrm{g}}+\left(\frac{1-\text{ɱ}_{\beta}}{\text{ɱ}_{\beta}}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}}\cdot e^{i2\pi\left(\frac{1}{1+\left\{\left(\frac{1-\text{ƴ}_{\breve{\alpha}}}{\text{ƴ}_{\breve{\alpha}}}\right)^{\mathrm{g}}+\left(\frac{1-\text{ƴ}_{\beta}}{\text{ƴ}_{\beta}}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}}\right)}, \\ 1-\frac{1}{1+\left\{\left(\frac{\dot{C}_{\breve{\alpha}}}{1-\dot{C}_{\breve{\alpha}}}\right)^{\mathrm{g}}+\left(\frac{\dot{C}_{\beta}}{1-\dot{C}_{\beta}}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}}\cdot e^{i2\pi\left(1-\frac{1}{1+\left\{\left(\frac{\text{ɖ}_{\breve{\alpha}}}{1-\text{ɖ}_{\breve{\alpha}}}\right)^{\mathrm{g}}+\left(\frac{\text{ɖ}_{\beta}}{1-\text{ɖ}_{\beta}}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}}\right)} \end{pmatrix}$$

2. $$N_a \otimes N_z = \begin{pmatrix} \frac{1}{1+\left\{\left(\frac{1-Ʒ_{\breve{a}}}{Ʒ_{\breve{a}}}\right)^{g}+\left(\frac{1-Ʒ_{\beta}}{Ʒ_{\beta}}\right)^{g}\right\}^{\frac{1}{g}}} . e^{i2\pi\left(\frac{1}{1+\left\{\left(\frac{1-\varphi_{\breve{a}}}{\varphi_{\breve{a}}}\right)^{g}+\left(\frac{1-\varphi_{\beta}}{\varphi_{\beta}}\right)^{g}\right\}^{\frac{1}{g}}}\right)}, \\ 1-\frac{1}{1+\left\{\left(\frac{ɱ_{\breve{a}}}{1-ɱ_{\breve{a}}}\right)^{m}+\left(\frac{ɱ_{\beta}}{1-ɱ_{\beta}}\right)^{g}\right\}^{\frac{1}{g}}} . e^{i2\pi\left(1-\frac{1}{1+\left\{\left(\frac{ɣ_{\breve{a}}}{1-ɣ_{\breve{a}}}\right)^{g}+\left(\frac{ɣ_{\beta}}{1-ɣ_{\beta}}\right)^{g}\right\}^{\frac{1}{g}}}\right)}, \\ \frac{1}{1+\left\{\left(\frac{1-\dot{C}_{\breve{a}}}{\dot{C}_{\breve{a}}}\right)^{g}+\left(\frac{1-\dot{C}_{\beta}}{\dot{C}_{\beta}}\right)^{g}\right\}^{\frac{1}{g}}} . e^{i2\pi\left(1-\frac{1}{1+\left\{\left(\frac{1-ɗ_{\breve{a}}}{ɗ_{\breve{a}}}\right)^{g}+\left(\frac{1-ɗ_{\beta}}{ɗ_{\beta}}\right)^{g}\right\}^{\frac{1}{g}}}\right)} \end{pmatrix}$$

3. $$\xi N_a = \begin{pmatrix} 1-\frac{1}{1+\left\{\xi\left(\frac{Ʒ_{\breve{a}}}{1-Ʒ_{\breve{a}}}\right)^{g}\right\}^{\frac{1}{g}}} . e^{i2\pi\left(1-\frac{1}{1+\left\{\xi\left(\frac{\varphi_{\breve{a}}}{1-\varphi_{\breve{a}}}\right)^{g}\right\}^{\frac{1}{g}}}\right)}, \\ \frac{1}{1+\left\{\xi\left(\frac{1-ɱ_{\breve{a}}}{ɱ_{\breve{a}}}\right)^{g}\right\}^{\frac{1}{g}}} . e^{i2\pi\left(\frac{1}{1+\left\{\xi\left(\frac{1-ɣ_{\breve{a}}}{ɣ_{\breve{a}}}\right)^{g}\right\}^{\frac{1}{g}}}\right)}, \\ 1-\frac{1}{1+\left\{\xi\left(\frac{\dot{C}_{\breve{a}}}{1-Ʒ_{\breve{a}}}\right)^{g}\right\}^{\frac{1}{g}}} . e^{i2\pi\left(1-\frac{1}{1+\left\{\xi\left(\frac{ɗ_{\breve{a}}}{1-ɗ_{\breve{a}}}\right)^{g}\right\}^{\frac{1}{g}}}\right)} \end{pmatrix}$$

4. $$N_a{}^{\xi} = \begin{pmatrix} \frac{1}{1+\left\{\xi\left(\frac{1-Ʒ_{\breve{a}}}{Ʒ_{\breve{a}}}\right)^{g}\right\}^{\frac{1}{g}}} . e^{i2\pi\left(\frac{1}{1+\left\{\xi\left(\frac{1-\varphi_{\breve{a}}}{\varphi_{\breve{a}}}\right)^{g}\right\}^{\frac{1}{g}}}\right)}, \\ 1-\frac{1}{1+\left\{\xi\left(\frac{ɱ_{\breve{a}}}{1-ɱ_{\breve{a}}}\right)^{g}\right\}^{\frac{1}{g}}} . e^{i2\pi\left(1-\frac{1}{1+\left\{\xi\left(\frac{ɣ_{\breve{a}}}{1-ɣ_{\breve{a}}}\right)^{g}\right\}^{\frac{1}{g}}}\right)}, \\ \frac{1}{1+\left\{\xi\left(\frac{1-\dot{C}_{\breve{a}}}{\dot{C}_{\breve{a}}}\right)^{g}\right\}^{\frac{1}{g}}} . e^{i2\pi\left(\frac{1}{1+\left\{\xi\left(\frac{1-ɗ_{\breve{a}}}{ɗ_{\breve{a}}}\right)^{g}\right\}^{\frac{1}{g}}}\right)} \end{pmatrix}$$

Definition 6. Consider $N_a = \left(Ʒ_a(r)e^{i2\pi\varphi_a(r)}, ɱ_a(r)e^{i2\piɣ_a(r)}, \dot{C}_a(r)e^{i2\piɗ_a(r)}\right)$ $(a = 1,2,\ldots,n')$ be C-CrIFVs. So, the C-CrIFDPWA operator is the function. $a^n \rightarrow a$ such that:

$$\text{C-CrIFDPWA}(N_1, N_2, \dots, N_{n'}) = \oplus_{a=1}^{n'} \text{ff}_a N_a,$$

Where $\text{ff} = (\text{ff}_1, \text{ff}_2, \dots, \text{ff}_a)^t$ and $\text{ff}_a = \frac{\Gamma_a}{\sum_{a=1}^{n'} \Gamma_a}$, $\Gamma_a = \prod_{a-1}^{n'} \mathring{A}(N_a)$ and $\Gamma_1 = 1$ be the weight vector (WV) of $N_a(a = 1,2, \dots, n')$, $\text{ff}_a > 0$, and $\sum_{a=1}^{n'} \text{ff}_i = 1$.

Theorem 1. Consider $N_a = \left(\mathfrak{Z}_a(r) e^{i2\pi \varphi_a(r)}, \mathfrak{m}_a(r) e^{i2\pi y_a(r)}, \dot{C}_a(r) e^{i2\pi d_a(r)}\right)$ $(a = 1,2, \dots, n')$ be some C-CrIFVs, and so, the combined value of them with C-CrIFDPWA process is also C-CrIFVs.

$$C - CrIFDPWA(N_1, N_2, \dots, N_{n'}) = \oplus_{i=1}^{n'} \text{ff}_a N_a$$

$$= \begin{pmatrix} \left(1 - \frac{1}{1 + \left\{\sum_{a=1}^{n'} \text{ff}_a \left(\frac{\mathfrak{Z}_a}{1 - \mathfrak{Z}_a}\right)^{g}\right\}^{\frac{1}{g}}}\right) . e^{i2\pi\left(1 - \frac{1}{1 + \left\{\sum_{a=1}^{n'} \text{ff}_a \left(\frac{\varphi_a}{1 - \varphi_a}\right)^{g}\right\}^{\frac{1}{g}}}\right)} \\ \frac{1}{1 + \left\{\sum_{a=1}^{n'} \text{ff}_a \left(\frac{1 - \mathfrak{m}_a}{\mathfrak{m}_a}\right)^{g}\right\}^{\frac{1}{g}}} . e^{i2\pi\left(\frac{1}{1 + \left\{\sum_{a=1}^{n'} \text{ff}_a \left(\frac{1 - y_a}{y_a}\right)^{g}\right\}^{\frac{1}{g}}}\right)} \\ \left(1 - \frac{1}{1 + \left\{\sum_{a=1}^{n'} \text{ff}_a \left(\frac{\dot{C}_a}{1 - \dot{C}_a}\right)^{g}\right\}^{\frac{1}{g}}}\right) . e^{i2\pi\left(1 - \frac{1}{1 + \left\{\sum_{a=1}^{n'} \text{ff}_a \left(\frac{d_a}{1 - d_a}\right)^{g}\right\}^{\frac{1}{g}}}\right)} \end{pmatrix}$$

Where $\text{ff} = (\text{ff}_1, \text{ff}_2, \dots, \text{ff}_a)^t$ be the WV of $N_a(a = 1,2, \dots, n')$, $\text{ff}_a > 0$ and $\sum_{a=1}^{n'} \text{ff}_a = 1$.

Proof. The proof of this theorem is given in the Appendix.

Theorem 2. (Idempotency property) Consider $N_a = \left(\mathfrak{Z}_a e^{i2\pi \varphi_a}, \mathfrak{m}_a e^{i2\pi y_a}, \dot{C}_a e^{i2\pi d_a}\right)$ $(a = 1,2, \dots, n')$ be C-CrIFVs are equal, i.e., $N_a = N$ for all a, where $N = (\mathfrak{Z}, \mathfrak{m}, \dot{C})$. Then:

$$C - CrIFDPWA\ (N_1, N_2, \dots, N_{n'}) = N.$$

Proof. The proof of this theorem is given in the Appendix.

Theorem 3. (Boundedness property) Consider $N_a = \left(\mathfrak{Z}_a e^{i2\pi \varphi_a}, \mathfrak{m}_a e^{i2\pi y_a}, \dot{C}_a e^{i2\pi d_a}\right)$ $(a = 1,2, \dots, n')$ be some C-CrIFVs. Consider $N^- = min\{N_1, N_2, \dots, N_{n'}\}$ and $N^+ = max\{N_1, N_2, \dots, N_{n'}\}$. Then

$$N^- \leq C - CrIFDPWA(N_1, N_2, \dots, N_{n'}) \leq N^+.$$

Proof: The proof of this theorem is given in the Appendix.

Theorem 4. (Monotonicity property) Consider N_a and N_a' be the collection of C-CrIFS for all a. So,

$$\text{C} - \text{CrIFDPWA}(N_1, N_2, \dots, N_{n'}) \geq \text{C} - \text{CrIFDPWA}\left(N_1', N_2', \dots, N_{n'}'\right).$$

Proof: The proof of this theorem is given in the Appendix.

Definition 7. Consider $N_a = \left(\mathfrak{Z}_a(r) e^{i2\pi \varphi_a(r)}, \mathfrak{m}_a(r) e^{i2\pi y_a(r)}, \dot{C}_a(r) e^{i2\pi d_a(r)}\right)$ $(a = 1,2, \dots, n')$ be C-CrIFVs. So, the C-CrIFDPWG operator is the function. $a^n \rightarrow a$ such that:

$$C-CrIFDPWG(N_1,N_2,\dots,N_{n'})=\otimes_{a=1}^{n'}(N_a)^{\text{ff}_a},$$

where $\text{ff}=(\text{ff}_1,\ \text{ff}_2,\dots,\ \text{ff}_a)^t$be the weight vector (WV) of $N_a(a=1,2,\dots,n'),\text{ff}_a>0,$ and $\sum_{a=1}^{n'}\text{ff}_i=1$.

Theorem 5. Consider $N_a=\left(\mathfrak{Z}_a(r)e^{i2\pi\varphi_a(r)},\mathfrak{m}_a(r)e^{i2\pi y_a(r)},\dot{C}_a(r)e^{i2\pi d_a(r)}\right)$ $(a=1,2,\dots,n')$ be some C-CrIFVs, so their combined value with the C-CrIFDPWG process is also C-CrIFVs.

$$C-CrIFDPWG(N_1,N_2,\dots,N_{n'})=\otimes_{a=1}^{n'}(N_a)^{\text{ff}_a}$$

$$=\left(\begin{array}{c}\dfrac{1}{1+\left\{\sum_{a=1}^{n'}\text{ff}_a\left(\dfrac{1-\mathfrak{Z}_a}{\mathfrak{Z}_a}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}}\cdot e^{i2\pi\left(\frac{1}{1+\left\{\sum_{a=1}^{n'}\text{ff}_a\left(\frac{1-\varphi_a}{\varphi_a}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}}\right)}\\ 1-\dfrac{1}{1+\left\{\sum_{a=1}^{n'}\text{ff}_a\left(\dfrac{\mathfrak{m}_a}{1-\mathfrak{m}_a}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}}\cdot e^{i2\pi\left(1-\frac{1}{1+\left\{\sum_{a=1}^{n'}\text{ff}_a\left(\frac{y_a}{1-y_a}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}}\right)}\\ \dfrac{1}{1+\left\{\sum_{a=1}^{n'}\text{ff}_a\left(\dfrac{1-\dot{C}_a}{\dot{C}_a}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}}\cdot e^{i2\pi\left(\frac{1}{1+\left\{\sum_{a=1}^{n'}\text{ff}_a\left(\frac{1-d_a}{d_a}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}}\right)}\end{array}\right)$$

where $\text{ff}=(\text{ff}_1,\ \text{ff}_2,\dots,\ \text{ff}_a)^t$ be the WV of $N_a(a=1,2,\dots,\ n'),\text{ff}_a>0$ and $\sum_{a=1}^{n'}\text{ff}_a=1$.

Proof. This proof is similar to the proof of Theorem 1.

Theorem 6. (Idempotency property) Consider $N_a=\left(\mathfrak{Z}_a e^{i2\pi\varphi_a},\mathfrak{m}_a e^{i2\pi y_a},\dot{C}_a e^{i2\pi d_a}\right)$ $(a=1,2,\dots,n')$ be C-CrIFVs are equal, i.e., $N_a=N$ for all a, where $N=(\mathfrak{Z},\mathfrak{m},\dot{C})$. Then:

$$C-CrIFDPWG\ (N_1,N_2,\dots,N_{n'})=N.$$

Proof. This proof is similar to the proof of Theorem 2.

Theorem 7. (Boundedness property) Consider $N_a=\left(\mathfrak{Z}_a e^{i2\pi\varphi_a},\mathfrak{m}_a e^{i2\pi y_a},\dot{C}_a e^{i2\pi d_a}\right)$ $(a=1,2,\dots,n')$ be some C-CrIFVs. Consider $N^-=min\{N_1,N_2,\dots,N_{n'}\}$ and $N^+=max\{N_1,N_2,\dots,N_{n'}\}$. Then

$$N^-\le C-CrIFDPWG(N_1,N_2,\dots,N_{n'})\le N^+.$$

Proof: This proof is similar to the proof of Theorem 3.

Theorem 8. (Monotonicity property) Consider N_a and N'_a be the collection of C-CrIFS for all a. So,

$$\mathrm{C-CrIFDPWG}(N_1,N_2,\dots,N_{n'})\ge\mathrm{C-CrIFDPWG}\left(N'_1,N'_2,\dots,N'_{n'}\right).$$

Proof. This proof is similar to the proof of Theorem 4.

4. MADM ALGORITHM BASED ON PROPOSED AOS

This section explains a novel MADM problem of selecting better art communication, leveraging emotional intelligence, based on the investigation and explanation above. Let the collection of art communication leveraging emotional intelligence alternatives is $\widehat{\mathfrak{Q}}_i = \left(\widehat{\mathfrak{Q}}_1, \widehat{\mathfrak{Q}}_2, \dots, \widehat{\mathfrak{Q}}_n\right)$, where $\widehat{\mathfrak{Q}}_i$ denotes the alternative $\widehat{\mathfrak{Q}}_i$. Consider $\bar{\Upsilon}'_i = (\bar{\Upsilon}'_1, \bar{\Upsilon}'_2, \dots, \bar{\Upsilon}'_n)$ be an assembly of attributes. Assume that the prioritized weight vector set for the attributes is $\text{ff}_i = (\text{ff}_1, \text{ff}_2, \dots, \text{ff}_i)$, where ff_i signifies the weight or importance degree relative to attributes $\overline{\overline{\mathrm{K}}}_i$. All of the data of art communication leveraging emotional intelligence selection development is built on C-CrIFDPWA and C-CrIFDPWG. Where we take the Dombi parameter is $\mathrm{g} = 2$. The stages involved in selecting an alternative are defined next.

Step 1. Take information from the specialist and make a C-CrIFS decision matrix $\mathbb{L}_{i\times j}$.

Step 2. In this step, data is obtained from the specialist; if the data is cost- or benefit-type, all the information is converted to a single cost or benefit type.

Step 3. In this step, calculate the attribute weight vector using the prioritized weight model to obtain the prioritized weights.

Step 4. Use the C-CrIFDPWA and C-CrIFDPWG operators to combine the C-CrIFS for every alternative. $\widehat{\mathfrak{Q}}_i$ in the decision matrix $\mathbb{L}_{i\times j}$.

$$C-CrIFDPWA(N_1, N_2, \dots, N_{n'}) = \begin{pmatrix} 1 - \dfrac{1}{1 + \left\{\sum_{a=1}^{n'} \text{ff}_a \left(\dfrac{\text{ʒ}_a}{1-\text{ʒ}_a}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}} \cdot e^{i2\pi\left(1 - \frac{1}{1 + \left\{\sum_{a=1}^{n'} \text{ff}_a \left(\frac{\varphi_a}{1-\varphi_a}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}}\right)}, \\ \dfrac{1}{1 + \left\{\sum_{a=1}^{n'} \text{ff}_a \left(\dfrac{1-\text{ɱ}_a}{\text{ɱ}_a}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}} \cdot e^{i2\pi\left(\frac{1}{1 + \left\{\sum_{a=1}^{n'} \text{ff}_a \left(\frac{1-\text{ɣ}_a}{\text{ɣ}_a}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}}\right)} \\ 1 - \dfrac{1}{1 + \left\{\sum_{a=1}^{n'} \text{ff}_a \left(\dfrac{\dot{\mathrm{C}}_a}{1-\dot{\mathrm{C}}_a}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}} \cdot e^{i2\pi\left(1 - \frac{1}{1 + \left\{\sum_{a=1}^{n'} \text{ff}_a \left(\frac{\text{ɖ}_a}{1-\text{ɖ}_a}\right)^{\mathrm{g}}\right\}^{\frac{1}{\mathrm{g}}}}\right)} \end{pmatrix}$$

$$C-CrIFDPWG(N_1,N_2,\dots,N_{n'}) = \left(\begin{array}{c} \dfrac{1}{1+\left\{\sum_{a=1}^{n'} \text{ff}_a \left(\frac{1-\text{ʒ}_a}{\text{ʒ}_a}\right)^{\text{g}}\right\}^{\frac{1}{\text{g}}}} \\ .e^{i2\pi\left(\frac{1}{1+\left\{\sum_{a=1}^{n'} \text{ff}_a \left(\frac{1-\varphi_a}{\varphi_a}\right)^{\text{g}}\right\}^{\frac{1}{\text{g}}}}\right)}, \\ 1-\dfrac{1}{1+\left\{\sum_{a=1}^{n'} \text{ff}_a \left(\frac{\text{ɱ}_a}{1-\text{ɱ}_a}\right)^{\text{g}}\right\}^{\frac{1}{\text{g}}}} \\ .e^{i2\pi\left(1-\frac{1}{1+\left\{\sum_{a=1}^{n'} \text{ff}_a \left(\frac{\text{ƴ}_a}{1-\text{ƴ}_a}\right)^{\text{g}}\right\}^{\frac{1}{\text{g}}}}\right)} \\ \dfrac{1}{1+\left\{\sum_{a=1}^{n'} \text{ff}_a \left(\frac{1-\dot{\text{C}}_a}{\dot{\text{C}}_a}\right)^{\text{g}}\right\}^{\frac{1}{\text{g}}}} \\ .e^{i2\pi\left(\frac{1}{1+\left\{\sum_{a=1}^{n'} \text{ff}_a \left(\frac{1-\text{ɖ}_a}{\text{ɖ}_a}\right)^{\text{g}}\right\}^{\frac{1}{\text{g}}}}\right)} \end{array}\right)$$

Step 5. Using Definition 2, compute the score value of the total aggregated results.

$$\text{Ḁ}(N) = \left(\frac{1}{2}\right)\left(\text{ʒ}(r)+\varphi(r)\right)-\left(\text{ɱ}(r)+\text{ƴ}(r)\right)\times\left(\dot{\text{C}}(r)-\text{ɖ}(r)\right).$$

Step 6. We rank the options to select the best one.

Step 7. End

Figure 2 illustrates the rough sketch of the proposed methodology. Construct a decision matrix using the C-CrIFVs and normalize it if necessary. Then, the established C-CrIFDPWA and C-CrIFDPWG models will be used to aggregate the decision matrix. Lastly, the alternatives are ranked by applying the score function formula.

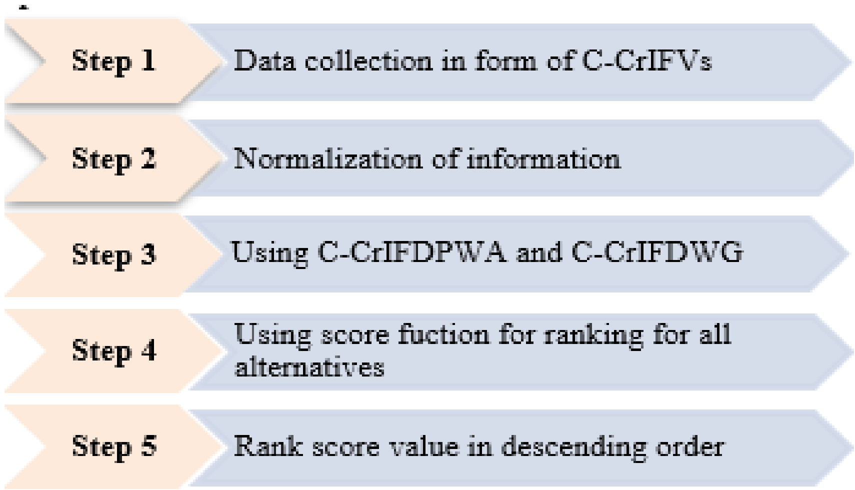

Fig. 2. Displays the proposed method

5. CASE STUDY

This study aims to identify the most effective emotionally intelligent system for enhancing art communication by evaluating multiple alternatives using MADM techniques. This case study integrates the C-CrIFS model, Dombi prioritized weighted averaging, prioritized weighted geometric operator, and the PROMETHEE technique to control emotional ambiguity and complex human-art interaction dynamics.

A current art museum seeks to implement a digital system to help create emotionally significant connections between visitors and artwork. With the post-COVID shift to virtual and engaging platforms, the organization aims to adopt an emotionally intelligent and artistically sensitive system to foster deeper engagement with diverse audiences.

5.1 Alternatives

In the following, we explain four alternatives; each handles emotionally intelligent communication in the art experience differently.

5.1.1 Emotionally Adaptive AI Interface

The emotionally adaptive AI interface adapts communication to users' emotional states and excels across numerous areas. Emotional Sensitivity is high in an emotionally adaptive AI interface because it uses emotion recognition tools, such as facial sentiment analysis, to personalize contacts. Its adaptability to artistic context may be moderate. Its essential function is user-focused rather than art-form-specific; it may need additional correction to suit different artistic formats, such as performance art. Its real-time interaction capability is generally strong, as AI models can adjust their responses to drift as they detect emotional changes.

Regarding the interpretability of emotional cues, this interface provides decent explanations of emotional reactions. Still, these are often simplified or generated from pre-trained models, which may not capture emotional nuance in complex artworks. Finally, its integration with the artistic medium is somewhat limited; while it can support the experience, it may feel more like a companion layer than a deeply embedded part of the artistic expression.

5.1.2 Human-Centered Artistic Feedback Loop

The human-centered artistic feedback loop emphasizes a two-way emotional exchange between artists and the audience, making it very strong in emotional Sensitivity. It allows authentic emotional reactions to directly influence artistic development, strengthening human connection. Its adaptability to artistic contexts is high, as it can be applied across many formats, including visual, performance, and digital art, especially when interaction is key. Its real-time interaction capability may be moderate. It excels at understanding emotional cues, relying on human Sensitivity and communication, and frequently provides richer, context-aware explanations. In the form of integration with artistic media, it performs well because emotional feedback becomes part of the creative process itself. However, scalability and consistency might be challenges in larger or digital contexts due to their less automated nature.

5.1.3 Multi-modal Emotion Recognition System

The multimodal emotion recognition system is built on the collection of emotional data from multiple sources, such as facial expressions, vocal tone, and biometric signals. This makes its emotional Sensitivity extremely high, as it does not rely on a single cue but cross-verifies across various channels to improve accuracy. Its adaptability to artistic context is also strong because it can be applied to many types of experiences, digital, live, or hybrid, with minimal adjustment. The system's real-time interaction capability is excellent when supported by high-performance computation and AI. Its understanding of emotional cues is high when it categorizes emotions clearly for the user or artist to understand and respond to. It also scores highly on integration with the artistic medium in tech-based art environments, where sensors and responsive connections are part of the installation. This strategy offers a balanced, highly effective approach to emotionally intelligent communication in artistic settings.

5.1.4 VR/AR-Based Empathic Art Experience

The VR/AR-based empathic art experience immerses users in collaborating with virtual environments that respond to their emotional states. Its emotional Sensitivity is moderate to high, depending on the sophistication of emotion-detection tools embedded within the VR/AR system. These systems can sometimes struggle with subtle shifts in expression, but Sensitivity improves when combined with haptics or physiological sensors. Its adaptability to artistic context is somewhat limited; although powerful in digital and attractive media, it does not easily translate to more traditional art spaces. The real-time interaction capability is a core strength of this alternative, as VR and AR environments are designed for immediate sensory feedback. However, the understandability of emotional cues may be weaker; users may feel the system is reacting to emotions but may not appreciate how or why those responses occur. Regarding integration with the artistic medium, this strategy outdoes tech-centric artworks but feels less organic in analog settings. It works best when the medium is the immersive technology itself.

5.2 Attributes

In the following, we provide a detailed explanation of each attribute used in the case study, highlighting its significance for fuzzy decision-making in art communication.

5.2.1 Emotional Sensitivity

Emotional sensitivity refers to the system's capability to accurately detect, interpret, and respond to users' emotional states in the art communication process. This attribute is essential because it determines if the system can identify understated sentimental cues, such as vocal tone shifts and body language changes. It creates more captivating and reliable knowledge, increases user satisfaction, and emotional engagement. Emotional Sensitivity is appropriate for fuzzy decision-making because it addresses vague human feelings that are best captured by the C-CrIF framework.

5.2.2 Adaptability to Artistic Context

Adaptability to artistic contexts is assessed to ensure the system can successfully work across various art frameworks. Artistic communication is inherently subjective and socially significant, making it essential that collaborating systems be flexible. This attribute is important in art communication because it measures how well the system understands the situation in which emotional expressions and reactions are embedded.

5.2.3 Real-time Interaction Capability

Real-time interaction capability measures the system's responsiveness, its ability to handle emotional content, and its capacity to produce significant outputs without suspension. Emotions shift vigorously in response to colors, movements, and connections; timing is critical in communication. Systems must capture emotional information and react in real time to maintain system engagement and the uninterrupted flow of emotion. Real-time performance is often difficult to estimate due to random information patterns, making it a vital attribute to model with the C-CrIF framework, addressing concerns about expectations and system feedback.

5.2.4 Interpretability of Emotional Cues

Interpretability of emotional cues is the system's ability to detect emotions and clearly convey what it understands from them. It supports the shot and hope in emotionally intelligent structures. Interpretability is a vital attribute in the art domain. It allows artists to understand how their work is being emotionally received. This attribute is critical when modeling with fuzzy sets because emotional data is often uncertain and context-dependent. Using C-CrIFS permits the system to present the strength of emotion and the hesitation and directional intent behind it.

5.3 Numerical example

This section applies the established C-CrIFDPWA and C-CrIFDPWG to real-world problems in selecting art communication that leverages emotional intelligence. We find better art communication leveraging emotional intelligence from the four alternatives: Emotionally Adaptive AI Interface. $\widehat{\mathfrak{Q}}_1$, Human-Centered Artistic Feedback Loop $\widehat{\mathfrak{Q}}_2$, Multi-modal Emotion Recognition System $\widehat{\mathfrak{Q}}_3$, VR/AR-Based Empathic Art Experience $\widehat{\mathfrak{Q}}_4$ shown as $\widehat{\mathfrak{Q}}_i$. We take four attributes such as $\bar{\Upsilon}'_1$ is Emotional Sensitivity, $\bar{\Upsilon}'_2$ is Adaptability to Artistic Context, $\bar{\Upsilon}'_3$ is Real-Time Interaction Capability and $\bar{\Upsilon}'_4$ is Interpretability of Emotional Cues. The prioritized weight of each attribute $\text{ff}_a = (\text{ff}_1, \text{ff}_2, \text{ff}_3, \text{ff}_4)$ is $\text{ff}_a = (0.53, 0.32, 0.11, 0.04)$ circulated by the expert. Next, data from experts in the form of C-CrIFS will be taken, and a decision matrix will be made. Where $\bar{\Upsilon}'_i$ represent the value of attributes and $\widehat{\mathfrak{Q}}_i$ denote the alternatives in the decision matrix. The decision matrix data is shown in Table 2.

Table 2

Decision matrix

$\widehat{\mathfrak{Q}}_i$	$\bar{\Upsilon}'_1$	$\bar{\Upsilon}'_2$	$\bar{\Upsilon}'_3$	$\bar{\Upsilon}'_4$
$\widehat{\mathfrak{Q}}_1$	$\begin{Bmatrix}(0.7, 0.3),\\(0.2, 0.1),\\(0.1, 0.2)\end{Bmatrix}$	$\begin{Bmatrix}(0.4, 0.1),\\(0.3, 0.5),\\(0.1, 0.2)\end{Bmatrix}$	$\begin{Bmatrix}(0.2, 0.4),\\(0.1, 0.4),\\(0.2, 0.3)\end{Bmatrix}$	$\begin{Bmatrix}(0.1, 0.4),\\(0.2, 0.5),\\(0.4, 0.3)\end{Bmatrix}$
$\widehat{\mathfrak{Q}}_2$	$\begin{Bmatrix}(0.3, 0.4),\\(0.6, 0.2),\\(0.2, 0.3)\end{Bmatrix}$	$\begin{Bmatrix}(0.6, 0.2),\\(0.2, 0.6),\\(0.2, 0.4)\end{Bmatrix}$	$\begin{Bmatrix}(0.2, 0.6),\\(0.2, 0.3),\\(0.4, 0.2)\end{Bmatrix}$	$\begin{Bmatrix}(0.4, 0.5),\\(0.3, 0.1),\\(0.2, 0.2)\end{Bmatrix}$
$\widehat{\mathfrak{Q}}_3$	$\begin{Bmatrix}(0.2, 0.5),\\(0.2, 0.1),\\(0.1, 0.2)\end{Bmatrix}$	$\begin{Bmatrix}(0.5, 0.4),\\(0.2, 0.4),\\(0.1, 0.3)\end{Bmatrix}$	$\begin{Bmatrix}(0.2, 0.4),\\(0.3, 0.5),\\(0.1, 0.4)\end{Bmatrix}$	$\begin{Bmatrix}(0.2, 0.4),\\(0.1, 0.3),\\(0.5, 0.3)\end{Bmatrix}$
$\widehat{\mathfrak{Q}}_4$	$\begin{Bmatrix}(0.4, 0.2),\\(0.5, 0.2),\\(0.2, 0.3)\end{Bmatrix}$	$\begin{Bmatrix}(0.3, 0.7),\\(0.6, 0.2),\\(0.5, 0.4)\end{Bmatrix}$	$\begin{Bmatrix}(0.1, 0.2),\\(0.2, 0.7),\\(0.2, 0.4)\end{Bmatrix}$	$\begin{Bmatrix}(0.5, 0.7),\\(0.4, 0.2),\\(0.3, 0.2)\end{Bmatrix}$

Table 2 represents C-CrIFS data, where each cell covers six elements: the first two elements indicate the satisfactory degree of real and imaginary parts, the following two terms show the dissatisfactory degree of real and imaginary parts, and the last two terms denote the degree of position allocated to the IFV.

We use the established model of C-CrIFDPWA and C-CrIFDPWG aggregation operation in Table 2, and combined results exist in Table 3.

Table 3

Aggregation finding by using C-CrIFDPWA and C-CrIFDPWG operators.

$\widehat{\mathfrak{Q}}_i$	**C-CrIFDPWA**	**C-CrIFDPWG**
$\widehat{\mathfrak{Q}}_1$	$\begin{Bmatrix}(0.6349, 0.2919),\\(0.1833, 0.1318),\\(0.1615, 0.2219)\end{Bmatrix}$	$\begin{Bmatrix}(0.2869, 0.1569),\\(0.2359, 0.3924),\\(0.1066, 0.2088)\end{Bmatrix}$
$\widehat{\mathfrak{Q}}_2$	$\begin{Bmatrix}(0.4785, 0.4243),\\(0.2697, 0.2207),\\(0.2443, 0.3329)\end{Bmatrix}$	$\begin{Bmatrix}(0.3117, 0.2833),\\(0.5255, 0.4680),\\(0.2081, 0.2898)\end{Bmatrix}$
$\widehat{\mathfrak{Q}}_3$	$\begin{Bmatrix}(0.3758, 0.4622),\\(0.1932, 0.1310),\\(0.1855, 0.2779)\end{Bmatrix}$	$\begin{Bmatrix}(0.2301, 0.4425),\\(0.2137, 0.3403),\\(0.1018, 0.2342)\end{Bmatrix}$
$\widehat{\mathfrak{Q}}_4$	$\begin{Bmatrix}(0.3668, 0.5858),\\(0.3864, 0.2094),\\(0.3774, 0.3350)\end{Bmatrix}$	$\begin{Bmatrix}(0.2248, 0.2375),\\(0.5303, 0.4472),\\(0.2335, 0.3205)\end{Bmatrix}$

To obtain a better alternative $\widehat{\mathfrak{Q}}_i$ we utilize the score function deliberated in Definition 2 among all combined outcomes. Outcomes are obtainable in Table 4, specified below:

Table 4

Scores value of combined data

$\widehat{\mathfrak{Q}}_i$	C-CrIFDPWA	C-CrIFDPWG
$\widehat{\mathfrak{Q}}_1$	-0.0185	0.0094
$\widehat{\mathfrak{Q}}_2$	-0.0183	0.0163
$\widehat{\mathfrak{Q}}_3$	-0.0237	-0.0079
$\widehat{\mathfrak{Q}}_4$	-0.0048	0.0224

For better consideration, in this step we use the score value to determine the ranking of the alternatives. The ranking of alternatives is shown in Table 5. The graphical representation of the score value is shown in Figure 3.

Table 5

Ranking of all alternatives

C-CrIFDPWA	$\widehat{\mathfrak{Q}}_4 > \widehat{\mathfrak{Q}}_2 > \widehat{\mathfrak{Q}}_1 > \widehat{\mathfrak{Q}}_3$
C-CrIFDPWG	$\widehat{\mathfrak{Q}}_4 > \widehat{\mathfrak{Q}}_2 > \widehat{\mathfrak{Q}}_1 > \widehat{\mathfrak{Q}}_3$

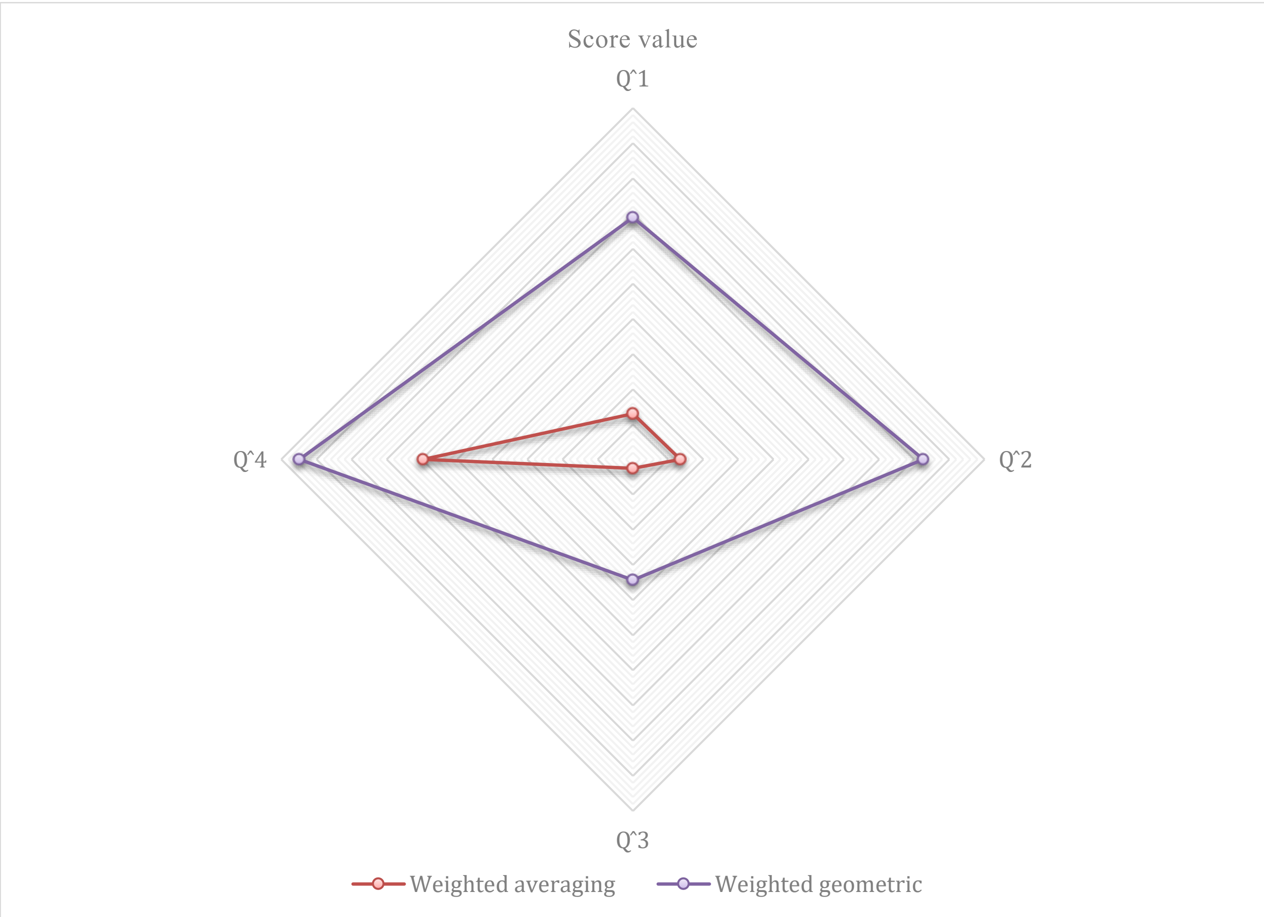

Fig. 3. Geometrical representation of score values.

In Figure 3 and Table 5, we observe that the multi-modal emotion recognition system is best. Based on the number of all attributes using the C-CrIFDPWA and C-CrIFDPWG, the Multi-modal Emotion recognition system is the best alternative for emotionally intelligent art communication.

5.4 PROMETHEE method

In this section, we explain the PROMETHEE method, which has been extended to various fuzzy frameworks, such as C-FS, interval-value fuzzy sets, generalized fuzzy sets, IFS, C-IFS, interval-value IFS, and C-IFS.

The PROMETHEE method is designed to address situations in which uncertainty arises in the best pairwise comparisons. This section explains the proposed method, which applies the PROMETHEE technique based on C-CrIFS. The steps of the PROMETHEE method are listed below:

Step 1: Assume that the expert assesses alternatives concerning each attribute using C-CrIFS. Then, make a decision matrix.

Step 2. When a decision matrix contains two attributes, beneficial and non-beneficial, normalization is required to make the data comparable. Attributes change into the same type by using the list below:

If the attributes are beneficial type:

$$Ƙ_{pq} = \frac{[a_{pq} - min(a_{pq})]}{[max(a_{pq}) - min(a_{pq})]}; (p = 1,2, \dots, n'; q = 1, 2, \dots, n').$$

For non-beneficial type attributes:

$$Ƙ_{pq} = \frac{[max(a_{pq}) - a_{pq}]}{[max(a_{pq}) - min(a_{pq})]}; (p = 1,2, \dots, n'; q = 1, 2, \dots, n').$$

Step 3. To calculate the assessing difference of the p^{th} alternative relative to the other options, measure how much it diverges from all other alternatives based on the normalized decision matrix. This is usually done by calculating the distance between the p^{th} alternative and each other. The result helps identify how distinct or preferable the p^{th} option is compared to the rest.

Step 4. Using the following equation to calculate the preference function $\mathcal{Q}(p, q)$ to prompt how much alternative p is ideal over the alternative q for a specific attribute. This function transforms the difference in performance between p and q into a value between 0 and 1, displaying the strength of preference.

$$\mathcal{Q}(p, q) = 0; \text{ if } Ƙ_{pq} < Ƙ_{pq}; \left(D\dot{m}_p - D\dot{m}_q\right) \leq 0,$$
$$\mathcal{Q}(p, q) = \left(Ƙ_{pq} - Ƙ_{pq}\right); \text{ If } Ƙ_{pq} > Ƙ_{pq}; \left(D\dot{m}_p - D\dot{m}_q\right) > 0.$$

Step 5. To use the listed formula, we combined the weighted preference function $ﬀ(p, q)$ separate preference values across all attributes using their prioritized weights. It exposes the complete preference for alternative p and q by combining weighted attributes-specific preferences:

$$ﬀ(p, q) = \sum_{j=1}^{\theta} ﬀ\, \mathcal{Q}(p, q).$$

Step 6. The entering flow m^{-} for the $\widehat{\mathfrak{Q}}_i$ Alternative measures how much all other alternatives rank p. It is calculated by averaging the weighted preference values $ﬀ(p, q)$ for all $p \neq q$ across all alternatives $\widehat{\mathfrak{Q}}$.

$$un^{-} = \frac{1}{\hat{\mathfrak{Q}} - 1} \sum_{i=1}^{n'} \mathrm{ff}(p, q).$$

as entering flow un^{+} for for $\hat{\mathfrak{Q}}_i$ alternative, described as follows:

$$un^{+} = \frac{1}{\hat{\mathfrak{Q}} - 1} \sum_{i=1}^{n'} \mathrm{ff}(p, q).$$

Step 7. The net ranking flow for each alternative $\hat{\mathfrak{Q}}_i$ is considered the difference between its entering flow un^{+} and entering flow un^{-}. It shows the overall intensity of an alternative by showing how much it ranks others and comparing how much it is ranked.

$$un = un^{+} - un^{-}.$$

Step 8. Finally, all alternatives $\hat{\mathfrak{Q}}_i$ are ordered under their net outranking flow values. The alternative with the highest net flow is measured as the most preferred or best choice.
All the steps of the algorithm are displayed in Figure 4.

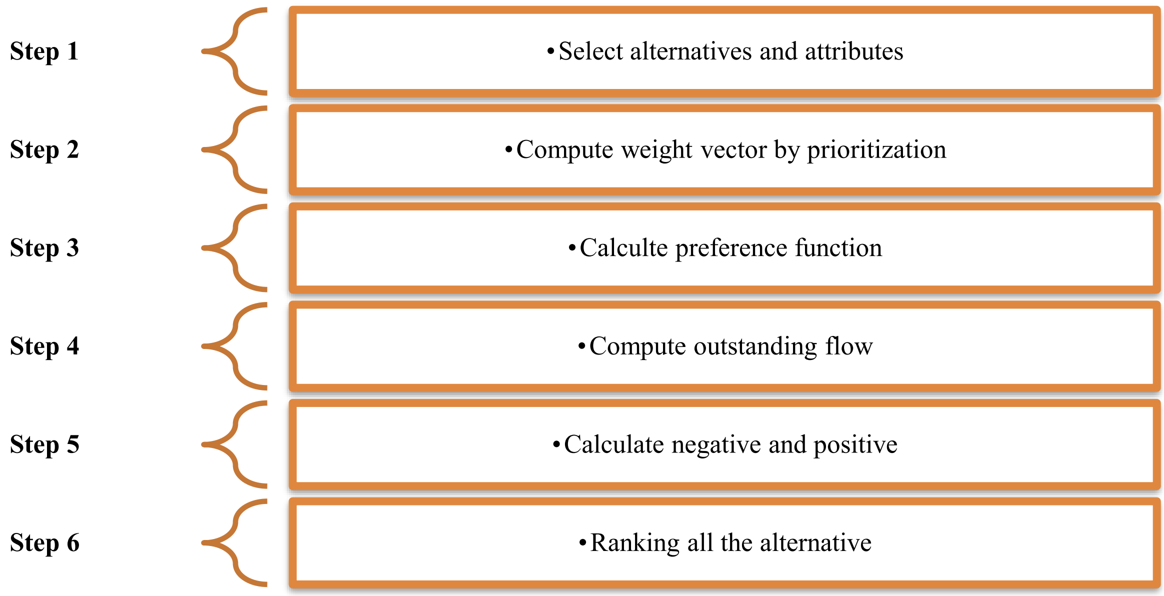

Fig. 4: Rough sketch of the steps of the PROMETHEE method

In Figure 4, we also explain the PROMETHEE model's step-by-step process. We can easily apply these steps to real-life situations. Now, we use all the steps to find the best alternative for emotionally intelligent art communication.

Step 1. Take a decision matrix under the C-CrIFVs exposed in Table 1.

Step 2. Rough sketch of the steps of the PROMETHEE method.

Step 3. Calculate the evaluating difference of $\hat{\mathfrak{Q}}_i$ alternative involving other options, all the aggregated values are given in Table 6.

Table 6

Evaluated difference of alternatives

	$\bar{\Upsilon}'_1$	$\bar{\Upsilon}'_2$	$\bar{\Upsilon}'_3$	$\bar{\Upsilon}'_4$
$(\hat{\mathfrak{Q}}_1-\hat{\mathfrak{Q}}_2)$	$\begin{pmatrix}(0.4,-0.1),\\(-0.4,-0.1)\\(-0.1,-0.1)\end{pmatrix}$	$\begin{pmatrix}(-0.2,-0.1),\\(0.1,-0.1),\\(-0.1,-0.2)\end{pmatrix}$	$\begin{pmatrix}(0,-0.2),\\(-0.1,0.1),\\(-0.2,0.1)\end{pmatrix}$	$\begin{pmatrix}(-0.3,-0.1),\\(-0.1,0.4),\\(0.2,0.1)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_1-\hat{\mathfrak{Q}}_3)$	$\begin{pmatrix}(0.5,-0.2),\\(0,0)\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(-0.1,-0.3),\\(0.1,0.1),\\(0,-0.1)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(-0.2,-0.1),\\(0.1,-0.1)\end{pmatrix}$	$\begin{pmatrix}(-0.1,0),\\(0.1,0.2),\\(-0.1,0)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_1-\hat{\mathfrak{Q}}_4)$	$\begin{pmatrix}(0.3,0.1),\\(-0.3,-0.1),\\(-0.1,-0.1)\end{pmatrix}$	$\begin{pmatrix}(0.1,-0.6),\\(-0.3,0.3),\\(-0.4,-0.2)\end{pmatrix}$	$\begin{pmatrix}(0.1\ 0.2),\\(-0.1,-0.3),\\(0,-0.1)\end{pmatrix}$	$\begin{pmatrix}(-0.4,-0.3),\\(-0.2,0.3),\\(0.1,0.1)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_2-\hat{\mathfrak{Q}}_1)$	$\begin{pmatrix}(-0.4,0.1),\\(0.4,0.1),\\(0.1,0.1)\end{pmatrix}$	$\begin{pmatrix}(0.2,0.1),\\(-0.1,0.1),\\(0.1,0.2)\end{pmatrix}$	$\begin{pmatrix}(0,0.2),\\(0.1,-0.1),\\(0.2,-0.1)\end{pmatrix}$	$\begin{pmatrix}(0.3,0.1),\\(0.1,-0.4),\\(-0.2,-0.1)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_2-\hat{\mathfrak{Q}}_3)$	$\begin{pmatrix}(0.1,-0.1),\\(0.4,0.1),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0.1,-0.2),\\(0,0.2),\\(0.1\ 0.1)\end{pmatrix}$	$\begin{pmatrix}(0,0.2),\\(-0.1,-0.2),\\(0.3,-0.2)\end{pmatrix}$	$\begin{pmatrix}(0.2,0.1),\\(0.2,-0.2),\\(-0.3,-0.1)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_2-\hat{\mathfrak{Q}}_4)$	$\begin{pmatrix}(-0.1,0.2),\\(0.1,0),\\(0.1,0.1)\end{pmatrix}$	$\begin{pmatrix}(0.3,-0.5),\\(-0.4,0.4),\\(-0.3,0)\end{pmatrix}$	$\begin{pmatrix}(0.1\ 0.4),\\(0,-0.4),\\(0.2,-0.2)\end{pmatrix}$	$\begin{pmatrix}(-0.1,-0.2),\\(-0.1,-0.1),\\(-0.1,0)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_3-\hat{\mathfrak{Q}}_1)$	$\begin{pmatrix}(-0.5,0.2),\\(0,0),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0.1,0.3),\\(-0.1,-0.1),\\(0,0.1)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0.2,0.1),\\(-0.1,0.1)\end{pmatrix}$	$\begin{pmatrix}(0.1,0),\\(-0.1,-0.2),\\(0.1,0)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_3-\hat{\mathfrak{Q}}_2)$	$\begin{pmatrix}(-0.1,0.1),\\(-0.4,-0.1),\\(-0.1,-0.1)\end{pmatrix}$	$\begin{pmatrix}(-0.1,0.2),\\(0,-0.2),\\(-0.1,-0.1)\end{pmatrix}$	$\begin{pmatrix}(0,-0.2),\\(0.1,0.2),\\(-0.3,0.2)\end{pmatrix}$	$\begin{pmatrix}(-0.2,-0.1),\\(-0.2,0.2),\\(0.3,0.1)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_3-\hat{\mathfrak{Q}}_4)$	$\begin{pmatrix}(-0.2,0.3),\\(-0.3,-0.1),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0.2,-0.3),\\(-0.4,0.2),\\(-0.4,-0.1)\end{pmatrix}$	$\begin{pmatrix}(0.1\ 0.2),\\(0.1,-0.2),\\(-0.1,0)\end{pmatrix}$	$\begin{pmatrix}(-0.3,-0.3),\\(-0.3,0.1),\\(0.2,0.1)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_4-\hat{\mathfrak{Q}}_1)$	$\begin{pmatrix}(-0.3,-0.1),\\(0.3,0.1),\\(0.1,0.1)\end{pmatrix}$	$\begin{pmatrix}(-0.1,0.6),\\(0.3,-0.3),\\(0.4,0.2)\end{pmatrix}$	$\begin{pmatrix}(-0.1,-0.2),\\(0.1,0.3),\\(0,0.1)\end{pmatrix}$	$\begin{pmatrix}(0.4,0.3),\\(0.2,-0.3),\\(-0.1,-0.1)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_4-\hat{\mathfrak{Q}}_2)$	$\begin{pmatrix}(0.1,-0.2),\\(-0.1,0),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(-0.3,0.5),\\(0.4,-0.4),\\(0.3,0)\end{pmatrix}$	$\begin{pmatrix}(-0.1,-0.4),\\(0.,0.4),\\(-0.2,0.2)\end{pmatrix}$	$\begin{pmatrix}(0.1,0.2),\\(0.1,0.1),\\(0.1,0)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_4-\hat{\mathfrak{Q}}_3)$	$\begin{pmatrix}(0.2,-0.3),\\(0.3,0.1),\\(0.1,0.1)\end{pmatrix}$	$\begin{pmatrix}(-0.2,0.3),\\(0.4,-0.2),\\(0.4,0.1)\end{pmatrix}$	$\begin{pmatrix}(-0.1-0.2),\\(-0.1,0.2),\\(0.1,0)\end{pmatrix}$	$\begin{pmatrix}(0.3,0.3),\\(0.3,-0.1),\\(-0.2,-0.1)\end{pmatrix}$

Step 4. In this step, we find the preference function values. The preference function is displayed in Table 7.

Table 7

Preference function values

	$\bar{\Upsilon}'_1$	$\bar{\Upsilon}'_2$	$\bar{\Upsilon}'_3$	$\bar{\Upsilon}'_4$
$(\hat{\mathfrak{Q}}_1 - \hat{\mathfrak{Q}}_2)$	$\begin{pmatrix}(0.4,0),\\(0,0)\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0.1,0),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0,0.1),\\(0,0.1)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0,0.4),\\(0.2,0.1)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_1 - \hat{\mathfrak{Q}}_3)$	$\begin{pmatrix}(0.5,0),\\(0,0)\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0.1,0.1),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0,0),\\(0.1,0)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0.1,0.2),\\(0,0)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_1 - \hat{\mathfrak{Q}}_4)$	$\begin{pmatrix}(0.3,0.1),\\(0,0),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0.1,0),\\(0,0.3),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0.1\ 0.2),\\(0,0),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0,0.3),\\(0.1,0.1)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_2 - \hat{\mathfrak{Q}}_1)$	$\begin{pmatrix}(0,0.1),\\(0.4,0.1),\\(0.1,0.1)\end{pmatrix}$	$\begin{pmatrix}(0.2,0.1),\\(0,0.1),\\(0.1,0.2)\end{pmatrix}$	$\begin{pmatrix}(0,0.2),\\(0.1,0),\\(0.2,0)\end{pmatrix}$	$\begin{pmatrix}(0.3,0.1),\\(0.1,0),\\(0,0)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_2 - \hat{\mathfrak{Q}}_3)$	$\begin{pmatrix}(0.1,0),\\(0.4,0.1),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0.1,0),\\(0,0.2),\\(0.1,0.1)\end{pmatrix}$	$\begin{pmatrix}(0,0.2),\\(0,0),\\(0.3,0)\end{pmatrix}$	$\begin{pmatrix}(0.2,0.1),\\(0.2,0),\\(0,0)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_2 - \hat{\mathfrak{Q}}_4)$	$\begin{pmatrix}(0,0.2),\\(0.1,0),\\(0.1,0.1)\end{pmatrix}$	$\begin{pmatrix}(0.3,0),\\(0,0.4),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0.1\ 0.4),\\(0,0),\\(0.2,0)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0,0),\\(0,0)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_3 - \hat{\mathfrak{Q}}_1)$	$\begin{pmatrix}(0,0.2),\\(0,0),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0.1,0.3),\\(0,0),\\(0,0.1)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0.2,0.1),\\(0,0.1)\end{pmatrix}$	$\begin{pmatrix}(0.1,0),\\(0,0),\\(0.1,0)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_3 - \hat{\mathfrak{Q}}_2)$	$\begin{pmatrix}(0,0.1),\\(0,0),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0.2),\\(0,0),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0.1,0.2),\\(0,0.2)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0,0.2),\\(0.3,0.1)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_3 - \hat{\mathfrak{Q}}_4)$	$\begin{pmatrix}(0,0.3),\\(0,0),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0.2,0),\\(0,0.2),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0.1\ 0.2),\\(0.1,0),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0,0.1),\\(0.2,0.1)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_4 - \hat{\mathfrak{Q}}_1)$	$\begin{pmatrix}(0,0),\\(0.3,0.1),\\(0.1,0.1)\end{pmatrix}$	$\begin{pmatrix}(0,0.6),\\(0.3,0),\\(0.4,0.2)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0.1,0.3),\\(0,0.1)\end{pmatrix}$	$\begin{pmatrix}(0.4,0.3),\\(0.2,0),\\(0,0)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_4 - \hat{\mathfrak{Q}}_2)$	$\begin{pmatrix}(0.1,0),\\(0,0),\\(0,0)\end{pmatrix}$	$\begin{pmatrix}(0,0.5),\\(0.4,0),\\(0.3,0)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0,0.4),\\(0,0.2)\end{pmatrix}$	$\begin{pmatrix}(0.1,0.2),\\(0.1,0.1),\\(0.1,0)\end{pmatrix}$
$(\hat{\mathfrak{Q}}_4 - \hat{\mathfrak{Q}}_3)$	$\begin{pmatrix}(0.2,0),\\(0.3,0.1),\\(0.1,0.1)\end{pmatrix}$	$\begin{pmatrix}(0,0.3),\\(0.4,0),\\(0.4,0.1)\end{pmatrix}$	$\begin{pmatrix}(0,0),\\(0,0.2),\\(0.1,0)\end{pmatrix}$	$\begin{pmatrix}(0.3,0.3),\\(0.3,0),\\(0,0)\end{pmatrix}$

Step 5. we combined weighted preference function Ж$(\mathfrak{p},\mathfrak{q})$ separate preference values across all attributes using their prioritized weights. Its outcomes are shown in Table 8.

Table 8

Aggregated weight preference function

	$\bar{\Upsilon}'_{\mathbf{1}}$	$\bar{\Upsilon}'_{\mathbf{2}}$	$\bar{\Upsilon}'_{\mathbf{3}}$	$\bar{\Upsilon}'_{\mathbf{4}}$
$\text{ff}_{\mathbf{1}}(\widehat{\mathfrak{Q}}_{\mathbf{1}} - \widehat{\mathfrak{Q}}_{\mathbf{2}})$	$\begin{pmatrix}(0.212, 0),\\(0, 0)\\(0, 0)\end{pmatrix}$	$\begin{pmatrix}(0, 0),\\(0.032, 0),\\(0, 0)\end{pmatrix}$	$\begin{pmatrix}(0, 0),\\(0, 0.011),\\(0, 0.011)\end{pmatrix}$	$\begin{pmatrix}(0, 0),\\(0, 0.016),\\(0.008, 0.004)\end{pmatrix}$
$\text{ff}_{\mathbf{1}}(\widehat{\mathfrak{Q}}_{\mathbf{1}} - \widehat{\mathfrak{Q}}_{\mathbf{3}})$	$\begin{pmatrix}(0.265, 0),\\(0, 0)\\(0, 0)\end{pmatrix}$	$\begin{pmatrix}(0, 0),\\(0.032, 0.032),\\(0, 0\ )\end{pmatrix}$	$\begin{pmatrix}(0, 0),\\(0, 0),\\(0.011, 0)\end{pmatrix}$	$\begin{pmatrix}(0, 0),\\(0.004, 0.008),\\(0, 0)\end{pmatrix}$
$\text{ff}_{\mathbf{1}}(\widehat{\mathfrak{Q}}_{\mathbf{1}} - \widehat{\mathfrak{Q}}_{\mathbf{4}})$	$\begin{pmatrix}(0.159, 0.053),\\(0, 0),\\(0, 0)\end{pmatrix}$	$\begin{pmatrix}(0.032, 0),\\(0, 0.096),\\(0, 0)\end{pmatrix}$	$\begin{pmatrix}(0.011\ 0.022),\\(0, 0),\\(0, 0)\end{pmatrix}$	$\begin{pmatrix}(0, 0),\\(0, 0.012),\\(0.004, 0.004)\end{pmatrix}$
$\text{ff}_{\mathbf{2}}(\widehat{\mathfrak{Q}}_{\mathbf{2}} - \widehat{\mathfrak{Q}}_{\mathbf{1}})$	$\begin{pmatrix}(0, 0.053),\\(0.212, 0.053),\\(0.053, 0.053)\end{pmatrix}$	$\begin{pmatrix}(0.064, 0.032),\\(0, 0.032),\\(0.032, 0.064)\end{pmatrix}$	$\begin{pmatrix}(0, 0.022),\\(0.011, 0),\\(0.022, 0)\end{pmatrix}$	$\begin{pmatrix}(0.012, 0.004),\\(0.004, 0),\\(0, 0)\end{pmatrix}$
$\text{ff}_{\mathbf{2}}(\widehat{\mathfrak{Q}}_{\mathbf{2}} - \widehat{\mathfrak{Q}}_{\mathbf{3}})$	$\begin{pmatrix}(0.053, 0),\\(0.212, 0.053),\\(0, 0)\end{pmatrix}$	$\begin{pmatrix}(0.032, 0),\\(0, 0.064),\\(0.032, 0.032)\end{pmatrix}$	$\begin{pmatrix}(0, 0.022),\\(0, 0),\\(0.033, 0)\end{pmatrix}$	$\begin{pmatrix}(0.008, 0.004),\\(0.008, 0),\\(0, 0)\end{pmatrix}$
$\text{ff}_{\mathbf{2}}(\widehat{\mathfrak{Q}}_{\mathbf{2}} - \widehat{\mathfrak{Q}}_{\mathbf{4}})$	$\begin{pmatrix}(0, 0.106),\\(0.053, 0),\\(0.053, 0.053)\end{pmatrix}$	$\begin{pmatrix}(0.096, 0),\\(0, 0.128),\\(0, 0)\end{pmatrix}$	$\begin{pmatrix}(0.011\ 0.044),\\(0, 0),\\(0.022, 0)\end{pmatrix}$	$\begin{pmatrix}(0, 0),\\(0, 0),\\(0, 0)\end{pmatrix}$
$\text{ff}_{\mathbf{3}}(\widehat{\mathfrak{Q}}_{\mathbf{3}} - \widehat{\mathfrak{Q}}_{\mathbf{1}})$	$\begin{pmatrix}(0, 0.106),\\(0, 0),\\(0, 0)\end{pmatrix}$	$\begin{pmatrix}(0.032, 0.096),\\(0, 0),\\(0, 0.032)\end{pmatrix}$	$\begin{pmatrix}(0, 0),\\(0.022, 0.011),\\(0, 0.011)\end{pmatrix}$	$\begin{pmatrix}(0.004, 0),\\(0, 0),\\(0.004, 0)\end{pmatrix}$
$\text{ff}_{\mathbf{3}}(\widehat{\mathfrak{Q}}_{\mathbf{3}} - \widehat{\mathfrak{Q}}_{\mathbf{2}})$	$\begin{pmatrix}(0, 0.053),\\(0, 0),\\(0, 0)\end{pmatrix}$	$\begin{pmatrix}(0, 0.064),\\(0, 0),\\(0, 0)\end{pmatrix}$	$\begin{pmatrix}(0, 0),\\(0.011, 0.022),\\(0, 0.022)\end{pmatrix}$	$\begin{pmatrix}(0, 0),\\(0, 0.008),\\(0.012, 0.004)\end{pmatrix}$
$\text{ff}_{\mathbf{3}}(\widehat{\mathfrak{Q}}_{\mathbf{3}} - \widehat{\mathfrak{Q}}_{\mathbf{4}})$	$\begin{pmatrix}(0, 0.159),\\(0, 0),\\(0, 0)\end{pmatrix}$	$\begin{pmatrix}(0.064, 0),\\(0, 0.064),\\(0, 0)\end{pmatrix}$	$\begin{pmatrix}(0.011\ 0.022),\\(0.011, 0),\\(0, 0)\end{pmatrix}$	$\begin{pmatrix}(0, 0),\\(0, 0.004),\\(0.008, 0.004)\end{pmatrix}$
$\text{ff}_{\mathbf{4}}(\widehat{\mathfrak{Q}}_{\mathbf{4}} - \widehat{\mathfrak{Q}}_{\mathbf{1}})$	$\begin{pmatrix}(0, 0),\\(0.159, 0.053),\\(0.053, 0.053)\end{pmatrix}$	$\begin{pmatrix}(0, 0.192),\\(0.096, 0),\\(0.128, 0.064)\end{pmatrix}$	$\begin{pmatrix}(0, 0),\\(0.011, 0.033),\\(0, 0.011)\end{pmatrix}$	$\begin{pmatrix}(0.016, 0.012),\\(0.008, 0),\\(0, 0)\end{pmatrix}$
$\text{ff}_{\mathbf{4}}(\widehat{\mathfrak{Q}}_{\mathbf{4}} - \widehat{\mathfrak{Q}}_{\mathbf{2}})$	$\begin{pmatrix}(0.053, 0),\\(0, 0),\\(0, 0)\end{pmatrix}$	$\begin{pmatrix}(0, 0.16),\\(0.128, 0),\\(0.096, 0)\end{pmatrix}$	$\begin{pmatrix}(0, 0),\\(0., 0.044),\\(0, 0.022)\end{pmatrix}$	$\begin{pmatrix}(0.004, 0.008),\\(0.004, 0.004),\\(0.004, 0)\end{pmatrix}$
$\text{ff}_{\mathbf{4}}(\widehat{\mathfrak{Q}}_{\mathbf{4}} - \widehat{\mathfrak{Q}}_{\mathbf{3}})$	$\begin{pmatrix}(0.106, 0),\\(0.159, 0.053),\\(0.053, 0.053)\end{pmatrix}$	$\begin{pmatrix}(0, 0.096),\\(0.128, 0),\\(0.128, 0.032)\end{pmatrix}$	$\begin{pmatrix}(0, 0),\\(0, 0.022),\\(0.011, 0)\end{pmatrix}$	$\begin{pmatrix}(0.012, 0.012),\\(0.012, 0),\\(0, 0)\end{pmatrix}$

Step 6. To consider the entering and leaving outranking, we obtain $ɯ^{-}$ for $\widehat{\mathfrak{Q}}_i$ alternative, and $ɯ^{+}$ for $\widehat{\mathfrak{Q}}_i$ alternative. Its outcomes are displayed in Table 9.

Table 10

Net outranking result

	$\bar{\Upsilon}'_1$	$\bar{\Upsilon}'_2$	$\bar{\Upsilon}'_3$	$\bar{\Upsilon}'_4$	$ɯ^+$
$\hat{\mathfrak{Q}}_1$	0	0.2940	0.3520	0.3930	0.3463
$\hat{\mathfrak{Q}}_2$	0.7230	0	0.5530	0.5660	0.6140
$\hat{\mathfrak{Q}}_3$	0.3180	0.1960	0	0.3470	0.2870
$\hat{\mathfrak{Q}}_4$	0.8890	0.5270	0.8770	0	0.7643
$ɯ^-$	0.6433	0.3390	0.5940	0.4353	

Step 8. Finally, we rank all the combined values to choose the best alternative. The ranking of all alternatives is shown in Table 11. We also display its graphical representation in Figure 5.

Table 11

Ranking of all alternatives

PROMETHEE method	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_2 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_3$

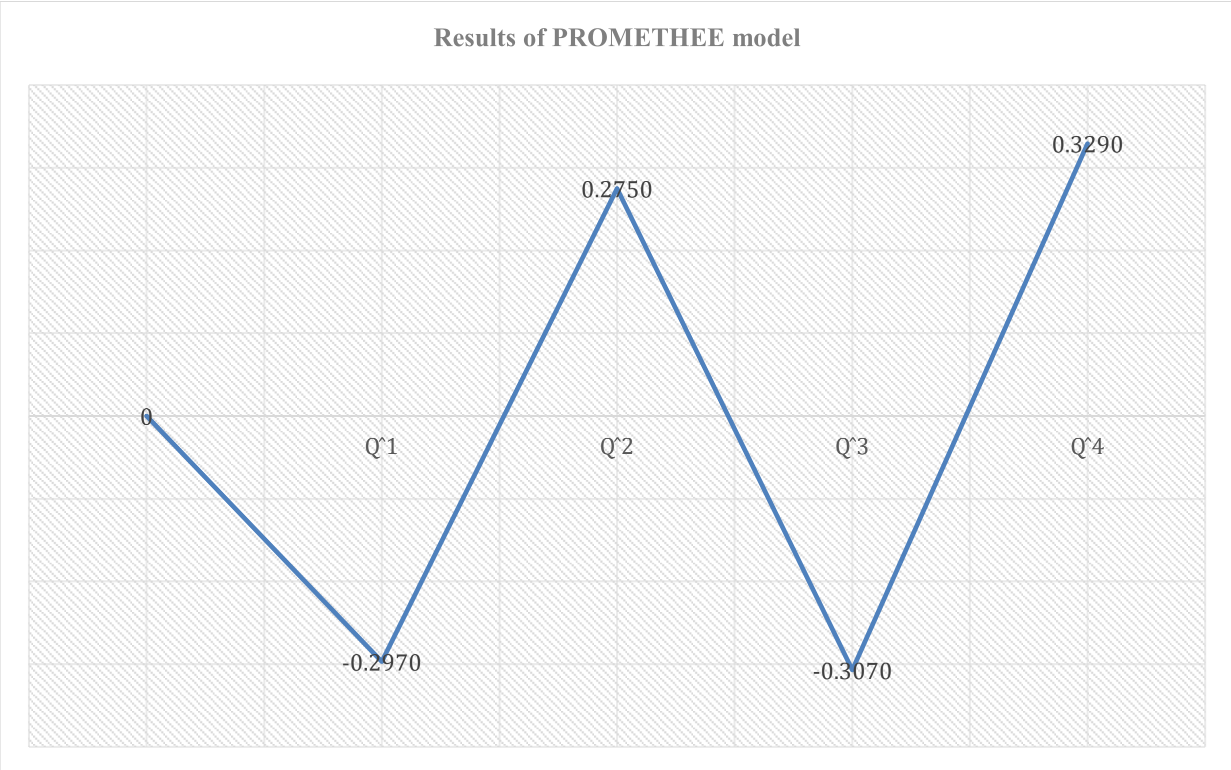

Fig. 5. The net outranking results

The outcomes reliably recognized the Multi-modal Emotion Recognition System as the most appropriate alternative, balancing emotional Sensitivity, adaptability, real-time responsiveness, interpretability, and medium integration.

6. COMPARATIVE ANALYSIS

This sector aims to investigate the influence of the radius term in C-CrIF Dombi aggregation operators. To show this, we take the decision matrix in Table 2 and aggregate it using the Complex circular intuitionistic fuzzy Dombi weighted averaging (C-CrIFDPWA) and the Complex circular intuitionistic fuzzy Dombi weighted geometric (C-CrIFDWG) proposed by Amin et al. [56] and eliminate the radius information from the decision matrix, then compare the decision matrix with complex intuitionistic fuzzy Dombi weighted averaging (C-IFDWA), complex intuitionistic fuzzy Dombi weighted geometric (C-IFDWG) invented by Masmali et al. [57], complex intuitionistic fuzzy Einstein weighted averaging (C-IFEWA), complex intuitionistic fuzzy Einstein weighted geometric (C-IFEWG) established by Rahman [58], complex intuitionistic fuzzy Frank weighted averaging (C-IFFWA), complex intuitionistic fuzzy Frank weighted geometric (C-IFFWG), invented by Yang et al. [59], complex intuitionistic fuzzy Aczel-Alsina weighted averaging (C-IFAAWA) established by Mahmood et al. [60], complex intuitionistic fuzzy Aczel-Alsina weighted geometric (C-IFAAWG). The effect of the radius on the combination results is then assessed by comparing the resultant accumulations. The comparative analysis ranking is displayed in Table 12, and the graphical representation of the score value is shown in Figures 6 and 7.

Table 12

Comparative analysis with existing AOs

Method	operator	Ranking
	C-CrIFDPWA	$\widehat{\mathfrak{Q}}_4 > \widehat{\mathfrak{Q}}_2 > \widehat{\mathfrak{Q}}_1 > \widehat{\mathfrak{Q}}_3$
	C-CrIFDPWG	$\widehat{\mathfrak{Q}}_4 > \widehat{\mathfrak{Q}}_2 > \widehat{\mathfrak{Q}}_1 > \widehat{\mathfrak{Q}}_3$
Proposed work	PROMETHEE method	$\widehat{\mathfrak{Q}}_4 > \widehat{\mathfrak{Q}}_2 > \widehat{\mathfrak{Q}}_1 > \widehat{\mathfrak{Q}}_3$
Amin et al. [56]	C-CrIFDWA	$\widehat{\mathfrak{Q}}_4 > \widehat{\mathfrak{Q}}_2 > \widehat{\mathfrak{Q}}_1 > \widehat{\mathfrak{Q}}_3$
	C-CrIFDWG	$\widehat{\mathfrak{Q}}_4 > \widehat{\mathfrak{Q}}_2 > \widehat{\mathfrak{Q}}_1 > \widehat{\mathfrak{Q}}_3$
Rahman [58]	C-IFEWA	$\widehat{\mathfrak{Q}}_1 > \widehat{\mathfrak{Q}}_4 > \widehat{\mathfrak{Q}}_2 > \widehat{\mathfrak{Q}}_3$
	C-IFEWG	$\widehat{\mathfrak{Q}}_1 > \widehat{\mathfrak{Q}}_4 > \widehat{\mathfrak{Q}}_2 > \widehat{\mathfrak{Q}}_3$
Masmali et al. [57]	C-IFDWA	$\widehat{\mathfrak{Q}}_1 > \widehat{\mathfrak{Q}}_3 > \widehat{\mathfrak{Q}}_2 > \widehat{\mathfrak{Q}}_4$
	C-IFDWG	$\widehat{\mathfrak{Q}}_3 > \widehat{\mathfrak{Q}}_1 > \widehat{\mathfrak{Q}}_2 > \widehat{\mathfrak{Q}}_4$
Yang et al. [59]	C-IFFWA	$\widehat{\mathfrak{Q}}_1 > \widehat{\mathfrak{Q}}_4 > \widehat{\mathfrak{Q}}_2 > \widehat{\mathfrak{Q}}_3$
	C-IFFWG	$\widehat{\mathfrak{Q}}_1 > \widehat{\mathfrak{Q}}_4 > \widehat{\mathfrak{Q}}_2 > \widehat{\mathfrak{Q}}_3$
Mahmood et al. [60]	C-IFAAWA	$\widehat{\mathfrak{Q}}_1 > \widehat{\mathfrak{Q}}_4 > \widehat{\mathfrak{Q}}_2 > \widehat{\mathfrak{Q}}_3$
	C-IFAAWG	$\widehat{\mathfrak{Q}}_4 > \widehat{\mathfrak{Q}}_1 > \widehat{\mathfrak{Q}}_2 > \widehat{\mathfrak{Q}}_3$
Rukhsar et al. [61]	IFWA	Fail to aggregate
	IFWG	Fail to aggregate
Ameer and Yousaf [62]	IFDPWA	Fail to aggregate
	IFDPWG	Fail to aggregate
Fahmi et al. [63]	IFHWA	Fail to aggregate
	IFHWG	Fail to aggregate

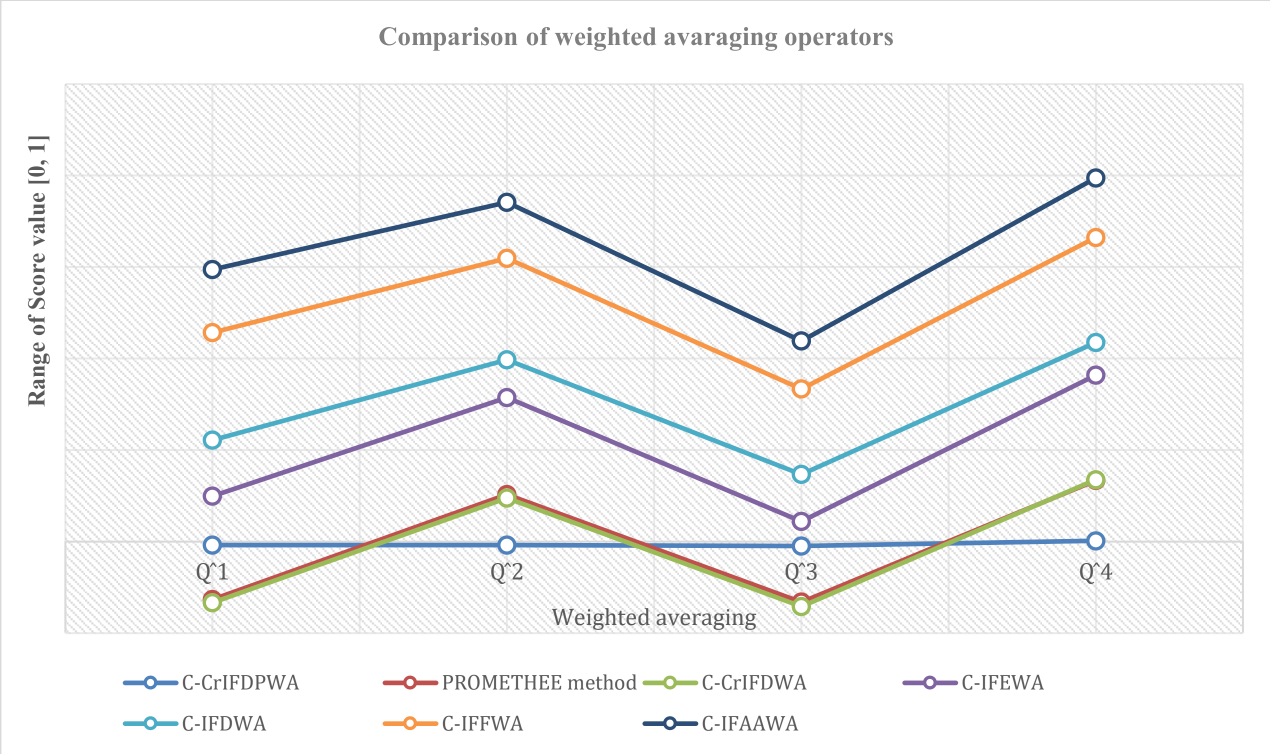

Fig. 6. Geometrical representation of a comparison of weighted averaging with other existing AOs.

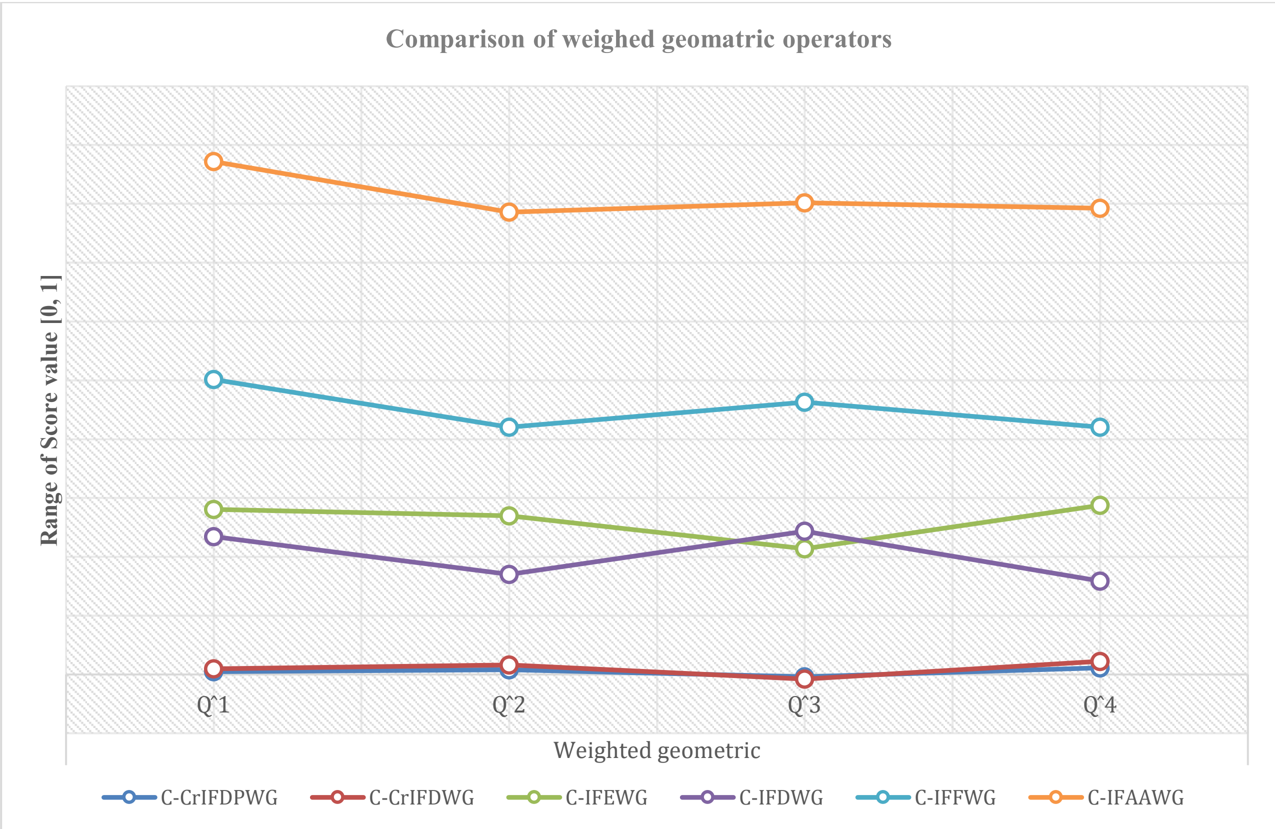

Fig. 7. Geometrical representation of a comparison of weighted geometric with other existing AOs.

In Figure 7, we observe that the score values exhibit significant variation as they develop from complex circular to standard complex intuitionistic fuzzy models by eliminating the radius term. The alterations based on the score of the radius and complex terms' vital impact on aggregation, especially regarding the rankings of alternatives $\hat{\mathfrak{Q}}_1$to $\hat{\mathfrak{Q}}_4$. This variation highlights how crucial these terms are for transmitting the depth and complexity of uncertainty in emotionally intelligent art communication evaluations.

6.1 Parametric analysis

In this section, we examine the Sensitivity of the parameter g by taking different values and using data from Table 2. The investigational outcomes using different parameter values are displayed in Tables 13 and 14 and graphically represented in Figures 8 and 9.

Table 13

Score value on different parameter values based on C-CrIFDPWA

	$\hat{\mathfrak{Q}}_1$	$\hat{\mathfrak{Q}}_2$	$\hat{\mathfrak{Q}}_3$	$\hat{\mathfrak{Q}}_4$	**Ranking**
$g = \mathbf{1}$	-0.0217	-0.012	-0.0296	-0.001	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_2 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_3$
$g = \mathbf{2}$	-0.0185	-0.0183	-0.0237	0.0046	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_2 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_3$
$g = \mathbf{3}$	-0.0087	-0.0198	-0.0097	0.0122	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_3 > \hat{\mathfrak{Q}}_2$
$g = \mathbf{4}$	0.0014	-0.0187	0.0018	0.0179	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_3 > \hat{\mathfrak{Q}}_2$
$g = \mathbf{5}$	0.0093	-0.0168	0.0096	0.0219	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_3 > \hat{\mathfrak{Q}}_2$
$g = \mathbf{6}$	0.0150	-0.0148	0.0150	0.0246	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_3 > \hat{\mathfrak{Q}}_2$
$g = \mathbf{7}$	0.0192	-0.0131	0.0188	0.0266	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_3 > \hat{\mathfrak{Q}}_2$
$g = \mathbf{8}$	0.0223	-0.0117	0.0217	0.0281	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_3 > \hat{\mathfrak{Q}}_2$

Table 14

Score value on different parameter values based on C-CrIFDPWG

	$\hat{\mathfrak{Q}}_1$	$\hat{\mathfrak{Q}}_2$	$\hat{\mathfrak{Q}}_3$	$\hat{\mathfrak{Q}}_4$	**Ranking**
$g = \mathbf{1}$	-0.0013	0.0101	-0.014	0.0112	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_2 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_3$
$g = \mathbf{2}$	0.0094	0.0163	-0.0079	0.0224	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_2 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_3$
$g = \mathbf{3}$	0.0155	0.0182	-0.0043	0.0291	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_2 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_3$
$g = \mathbf{4}$	0.0190	0.0179	-0.0020	0.0314	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_2 > \hat{\mathfrak{Q}}_3$
$g = \mathbf{5}$	0.0213	0.0167	-0.0003	0.0313	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_2 > \hat{\mathfrak{Q}}_3$
$g = \mathbf{6}$	0.0227	0.0153	0.0010	0.0299	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_2 > \hat{\mathfrak{Q}}_3$
$g = \mathbf{7}$	0.0238	0.0139	0.0021	0.0278	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_2 > \hat{\mathfrak{Q}}_3$
$g = \mathbf{8}$	0.0246	0.0126	0.0030	0.0256	$\hat{\mathfrak{Q}}_4 > \hat{\mathfrak{Q}}_1 > \hat{\mathfrak{Q}}_2 > \hat{\mathfrak{Q}}_3$

According to Tables 13 and 14, while the ranking of all alternatives for each parameter is not the same, the best outcomes are the same, the best alternative is $\hat{\mathfrak{Q}}_4$. To investigate the above parameter, it can be realized that the parameter g can replicate the inclinations of different experts so that experts can select compliantly according to various decision situations and risk preferences. For instance, risk-avoiding experts can select a lower value for the parameter. This is also supported by evidence that the C-CrIFDPWA and C-CrIFDPWG operator-based MADM techniques can provide precise confidence in the outcome.

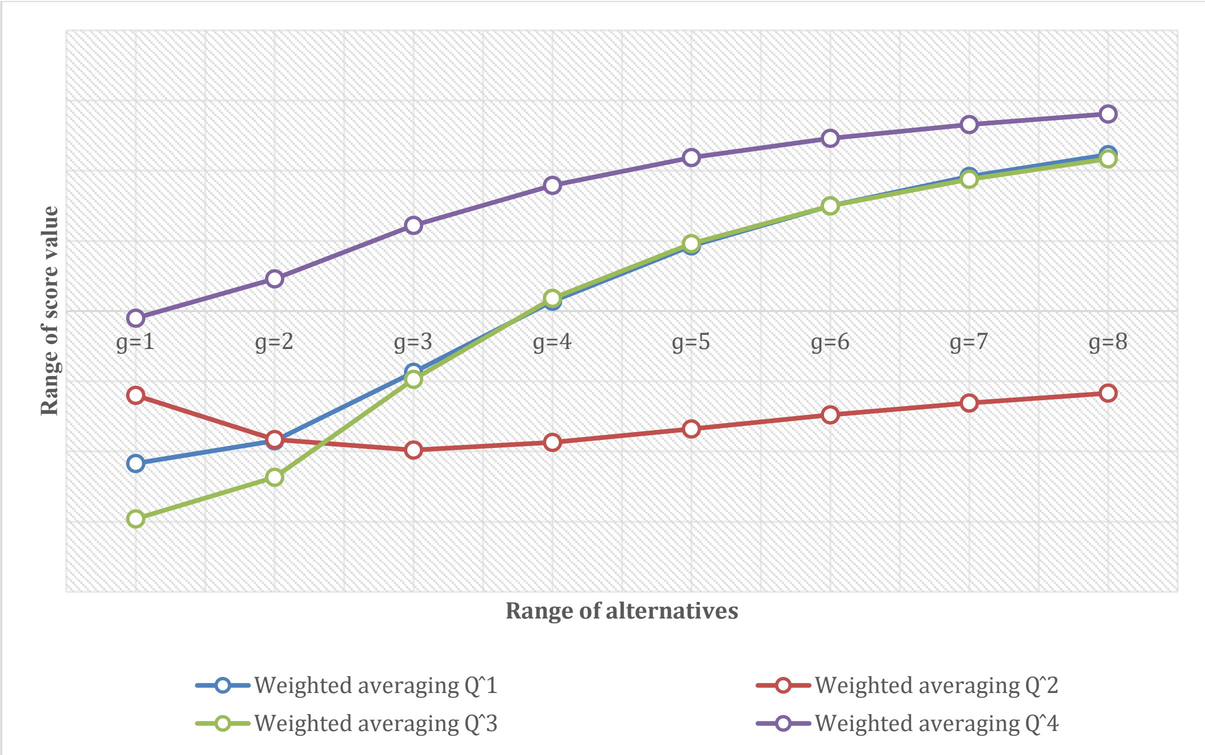

Fig. 8. Score value with different parameter values using C-CrIFDPWA

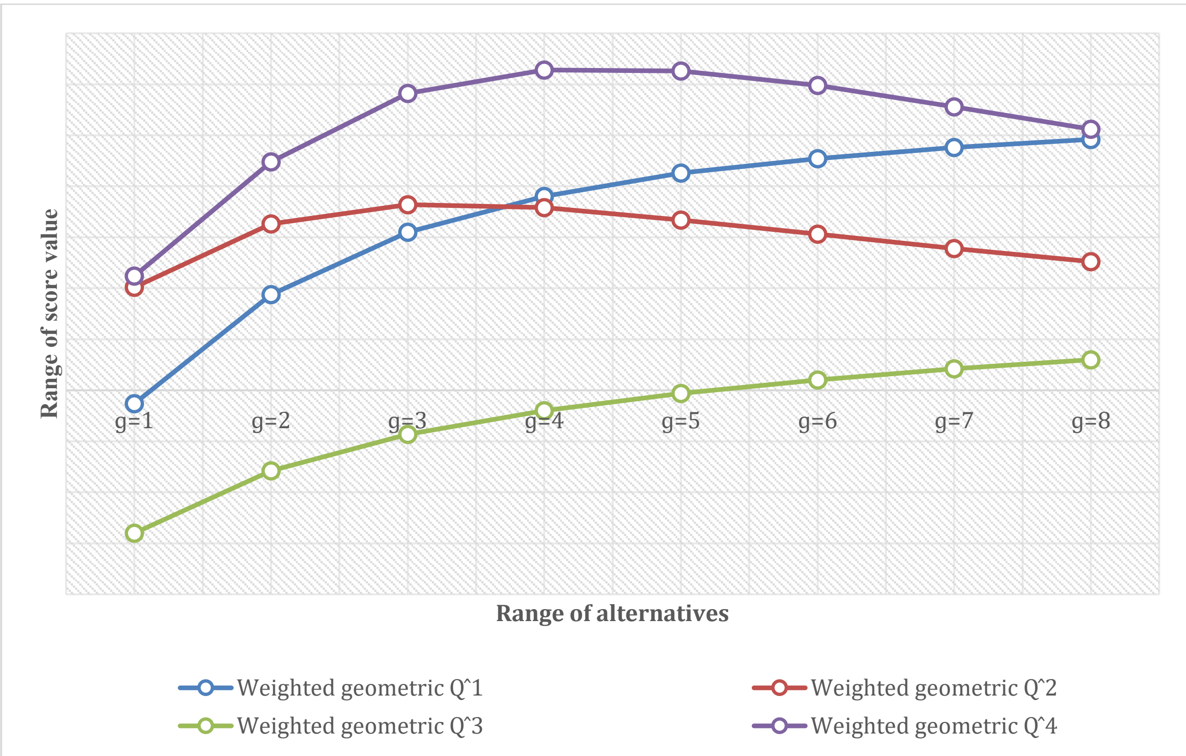

Fig. 9. Score value with different parameter values using C-CrIFDPWG

In Figures 8 and 9, we observe that the ranking of all alternatives for each parameter is not the same, but the best outcomes are the same, and the best alternative is $\widehat{\mathfrak{Q}}_4$.

6.3 Control of Emotionally Intelligent Art Communication through the Proposed Framework

The introduced approach governs emotionally intelligent art communication by incorporating C-CrIFS, Dombi aggregation operators, and the PROMETHEE method to establish models, analyze, and rank emotional exchanges under uncertainty and vagueness. The C-CrIFS framework enables the system to concurrently capture emotional sensitivity, hesitancy, displeasure, and directional emotional behavior in artistic communication. Additionally, the designed C-CrIFDPWA and C-CrIFDPWG operators enable the system to aggregate emotional data across various attributes and experts, while accounting for their relative importance in evaluating alternative methods of artistic communication. In addition, the Dombi operational rules offer sufficient flexibility to adjust decision-making behavior under varying emotions and risks. Moreover, the PROMETHEE approach oversees the final selection process by conducting pairwise comparisons and outranking analysis to determine the most emotionally productive communication scheme. This approach allows for intelligent assessment of audience engagement, emotional response, interpretability of emotional signals, adaptability to artistic communication context, and ability to interact in real time.

6.4 Practical implementation of the proposed model

6.4.1 Smart Education Systems

In a personalized education system, emotionally adaptive learning environments are important. Using our proposed model, C-CrIFS, innovative education programs can support real-time decision-making to tone, adapt lessons, and provide feedback in response to learners' emotional and intellectual states. For example, an effective tutor, prepared with emotional awareness and fuzzy decision-making, can determine whether to inspire, challenge, or comfort a student based on observed emotional responses such as frustration, joy, and uncertainty. Furthermore, our proposed PROMETHEE method helps determine the best ranking and assess many teaching plans for each student based on their learning style, emotional responses, and performance.

6.4.2 Human-Robot Interaction

Emotional intelligence is crucial for in-service robots to ensure safe, comforting, and efficient contact. Combining C-CrIFS with Dombi AOs, such as C-CrIFDPWA and C-CrIFDPWG, and the PROMETHEE method allows robots to evaluate emotional expressions and respond to needs. For example, a home-assistance robot can use this method to choose among multiple options, such as informing a caretaker, playing calming music, or engaging in conversation, by assessing the user's facial expressions and physiological signals. These proposed AOs handle ambiguity and uncertainty in emotional cues, which is essential in unpredictable, real-world environments where humans interact with non-human agents.

6.4.3 Military Strategy and Defense Planning

In defense applications, decision environments driven by C-CrIFS can support Threat investigation where intelligence information is uncertain or contradictory. Equipment obtaining decisions include cost, reliability, and battlefield effectiveness. Real-time mission support by ranking situations based on landscape, emotional stress levels, enemy activity, and resource availability. Our established

Dombi AOs and PROMETHEE method are ideal for combining contradictory priorities and dynamically adapting the ranking.

6.4.4 Cybersecurity and risk assessment

In cybersecurity and risk assessment, uncertainty evolution claims flexible decision-making models. C-CrIFS accurately captures indicators that are unclear, such as user behavior and trust levels. Our proposed Dombi AOs help prioritize attributes that are incompatible, such as threat severity vs. response urgency. C-CrIFS with the PROMETHEE method gives effective ranks of all alternatives under uncertainty. These tools enable more intelligent and adaptive risk management in dynamic cyber environments.

7. CONCLUSIONS

This investigation has effectively demonstrated the usefulness of C-CrIF Dombi aggregation operators for selecting emotionally intelligent art for communication. Our established model not only describes degrees of truth and falsity but also shows that the complex elements of radius and complex terms meaningfully affect the combination method. Also, it improves the decision-making structure. Dombi aggregation operations offer more rational and consistent outcomes than current aggregation operations. Prioritization is a useful method for calculating attribute weight vectors by combining them. Using the Dombi aggregation operator and C-CrIFS, we developed new aggregation operators, such as C-CrIFDPWA and C-CrIFDPWG, and proposed a PROMETHEE method based on C-CrIFS. These operators best manage the doubts and fuzziness inherent in emotionally intelligent art communication, thereby enabling extensive development beyond the traditional assessment approach. The outcomes reliably recognized the Multi-modal Emotion Recognition System as the most appropriate alternative, balancing emotional Sensitivity, adaptability, real-time responsiveness, interpretability, and medium integration. The pattern of C-CRIFS with prioritized and geometric analysis offered deep insight into the significance requirements of emotionally intelligent art communication. The improvements offered in this work lay a solid foundation for current developments in decision-making methods. This proposed model has substantially improved the justification of scientific novelty, strengthened the literature review with more critical discussion, included detailed sensitivity and comparative analyses, and corrected the editorial. All results from the proposed model significantly enhance the clarity, robustness, and overall quality of the study.

7.1 Future Directions

We can improve emotionally intelligent art communication and open novel ways for its application across numerous fields. In summary, our proposed model promotes further investigation of C-CrIF Dombi AOs within interactive art installations, AI-generated content, and digital curatorial systems, thereby pushing the boundaries of immersive, emotionally aware art experiences and expanding their wide-ranging applicability in decision-making under hesitation. Future research will expand this model by incorporating dynamic fuzzy cognitive maps to analyze temporal changes in audience emotion and artistic impact. Integrating neurasthenic data and AI-driven sentiment learning will improve real-time emotional interpretation. Moreover, implanting this fuzzy framework into the Pythagorean FS proposed by Asif et al. [64], Raja et al. [65] established q-rung orthopair fuzzy N-soft sets, Liu et al. [66] given security risk assessment, t-spherical FS for power AOs established by Khan et al. [67] and the notion of spherical fuzzy proposed by Mahmood et al. [68].

Credit authorship contribution statement
Muhammad Rizwan Khan proposed the C-CrIFDPWA and C-CrIFDPWG operators, as well as the PROMETHEE method, for assessing MADM problems. Kifayat Ullah compared it with other approaches to show the authenticity of the developed theory. Ali Raza writes the application and conclusion sections. Mijanur Rahaman Seikh handles all calculations for manuscripts.

Consent to participate
Each participant read and signed the consent form before the measurements.

Consent for publication
Each participant read and accepted the declaration of consent before the measurements and accepted that the data provided could be used and published in scientific form.

Code availability
Not applicable.

Foundation:
This research received no external funding.

Conflict of interest:
The authors declare no conflict of interest

Data availability
The datasets used and/or analyzed during the current study are available from the corresponding author upon reasonable request.

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